

# Secular Perturbations of Orbital Elements of Asteroids in the Thule Group

T.A. Vinogradova

**Abstract.** Secular perturbations of orbital elements of asteroids in the Thule group were studied. Asteroids of this group move in mean motion resonance 4:3 with Jupiter. The study was conducted using numerical integration of equations of motion for real asteroids. A simple dynamic model was used, taking into account perturbations from Jupiter, moving in a fixed elliptical orbit. After identifying classical secular perturbations, their influence on orbital elements was eliminated. This allowed us to identify the Lidov-Kozai mechanism in this resonant region. As a result, it was found that the maximum eccentricity, and correspondingly the minimum orbital inclination, are observed not at the usual values of the argument of perihelion  $\omega = 90^\circ, 270^\circ$ , but at the points  $\omega = 0^\circ, 180^\circ$ .

## Introduction

Secular perturbations of asteroids, both classical and Lidov-Kozai mechanism (LKM), are treated in the analytical theory by assuming that no mean-motion commensurabilities take place [1, 2, 3]. However, there are very strong mean motion resonances in the asteroid belt, where analytical methods cannot be applied.

The LKM in the region of resonances was studied by Y. Kozai [4] using a semi-analytical method. It was shown that in resonant regions the LKM manifests itself differently than in non-resonant ones. For the 4:3 resonance, i.e. in the Thule group, it was found that eccentricities  $e$  take maximum and inclinations  $i$  take minimum at  $2\omega = 0^\circ$  and vice versa at  $2\omega = 180^\circ$ .

Also, T. Vinogradova [5] used another method to study the LKM in Hildas and Trojans. The method is based on numerical integration of asteroid's equations of motion and elimination of classical secular perturbations. This method is used in the present work.

## 1. Method

The method used long-term numerical integration. An orbit evolution of real asteroids in the Thule group was investigated. As a source of initial osculating elements, the IAU Minor Planet Center (MPC) database of orbital elements version Nov. 2024 was employed. Elements of the Jupiter's orbit were fixed at the moment corresponding to the epoch of asteroid orbital elements.

At first, classical secular perturbations were identified. Then these perturbations were excluded from osculating elements. An exclusion of classical perturbations can be made very easy using a coordinate transformation formula. After excluding classical secular perturbations, LKM  $i$  and  $e$  oscillations can become visible in the distributions of proper elements.

## 2. Classical secular perturbations

In the case of classical secular perturbations,  $i$  variations depends on the longitude of the ascending node  $\Omega$ , and  $e$  variations – on the longitude of perihelion  $\varpi$ .

As an example, the motion of (185290) 2006UB219 from the Thule group was considered. The results of numerical integration in Figure 1 show well classical secular perturbations in the orbital elements of this asteroid.

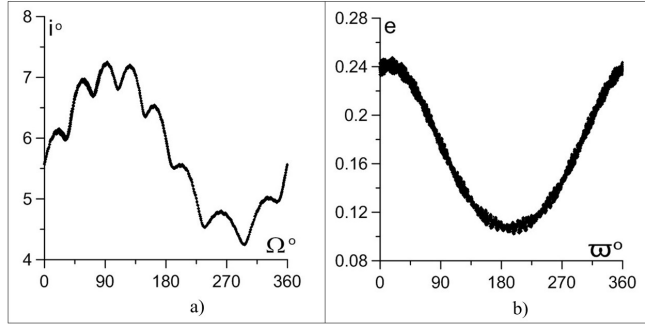


FIGURE 1. The results of numerical integration show variations of orbital elements of the asteroid (185290)2006 UB219 under the influence of classical secular perturbations:  $(i, \Omega)$  and  $(e, \varpi)$ .

## 3. Lidov-Kozai perturbations

The LKM is a secular perturbation which causes long-period oscillations in the inclination  $i$  and eccentricity  $e$  depending on a perihelion argument  $\omega$ . Under the influence of LKM,  $i$  and  $e$  change in concert.

After excluding classical secular perturbations, LKM  $i$  and  $e$  oscillations can become visible in distributions of proper elements  $(i_p, \omega_p)$  and  $(e_p, \omega_p)$ . These oscillations can be seen in Figure 2. One can see that usual positions of the maximum  $e$  and, accordingly, minimum  $i$  ( $\omega = 90^\circ, 270^\circ$ ) are shifted by  $90^\circ$  and is achieved with  $\omega = 0^\circ, 180^\circ$ .

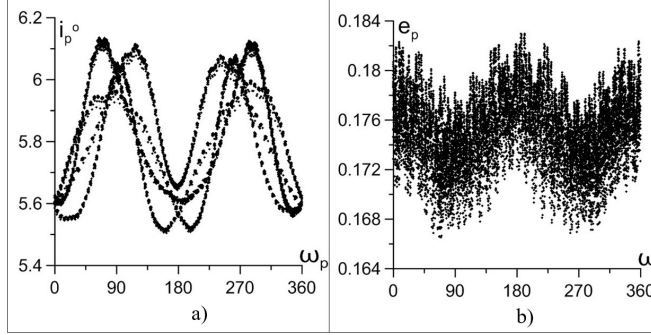


FIGURE 2. LKM oscillations of the orbital elements  $i$  and  $e$  of the asteroid (185290)2006 UB219 in the Thule group.

## Conclusion

The results by Y. Kozai [4], concerning LKM in resonance 4:3, are confirmed in this work using a different method.

## References

- [1] Brouwer D., van Woerkom A. J. J., *The secular variations of the orbital elements of the principal planets*. Astron. Papers Amer. Ephem., 1950.
- [2] Lidov M. L., *The evolution of orbits of artificial satellites of planets under the action of gravitational perturbations of external bodies*. Planetary and Space Science. Vol. 9 (10), 1962.
- [3] Kozai Y., *Secular perturbations of asteroids with high inclination and eccentricity*. The Astronomical Journal, Vol. 67, 1962.
- [4] Kozai Y., *Secular Perturbations of Resonant Asteroids*. Celestial Mechanics, Vol. 36, 1985.
- [5] Vinogradova T. A., *Lidov-Kozai mechanism in Hildas and Jupiter Trojans*. Celestial Mechanics, Vol. 136, 17, 2024.

T.A. Vinogradova

Institute of Applied Astronomy, Russian Academy of Sciences, St. Pe-tersburg, Russia  
e-mail: vta@iaaras.ru