

Secular Perturbations of Orbital Elements of Asteroids in the Thule Group

T.A. Vinogradova

*Institute of Applied Astronomy, Russian Academy of Sciences,
St. Petersburg, Russia*

Thule dynamical group

The asteroids orbit in the outermost edge of the asteroid belt in a 4:3 orbital resonance with Jupiter.

4.29 a.u. corresponds to the exact commensurability.

Asteroid 279 Thule was discovered in 1888y.

Diam = 127 km

Other members were discovered only in the 2000s.

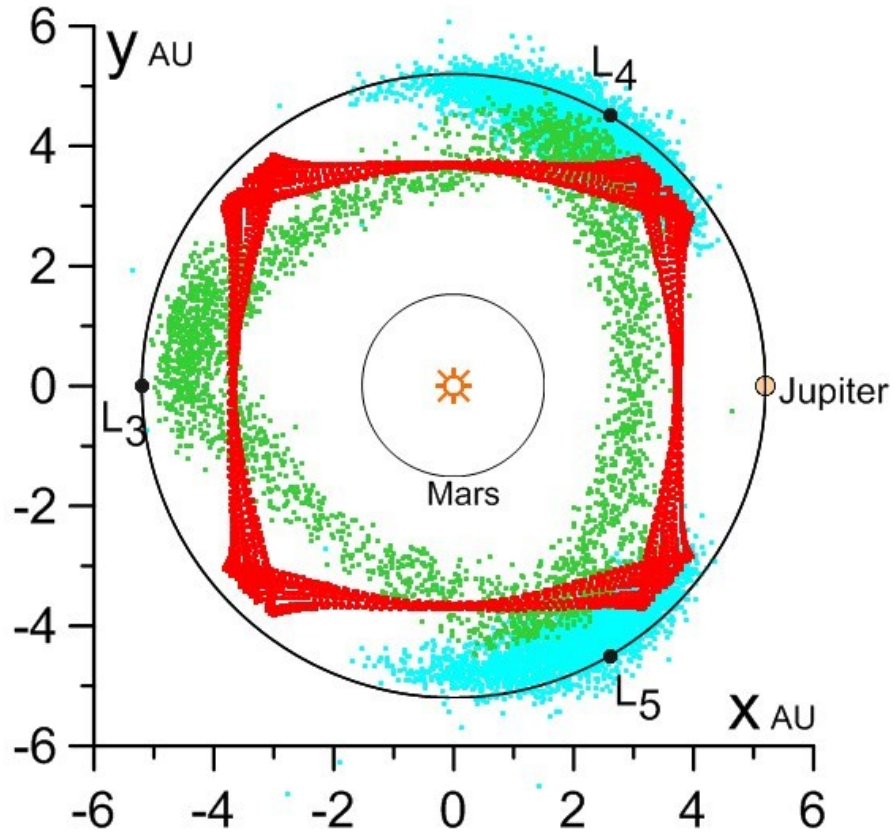
Name	a (a.u.)	e	i (deg)	H ^m
279 Thule	4.27	0.04	2.33°	8.6 ^m
185290 2006 UB219	4.30	0.13	7.1	13.9
186024 2001 QG207	4.29	0.25	3.2	14.6
570461 2006 SJ42	4.29	0.04	5.5	15.5
654117 2014 WN504	4.30	0.23	3.2	15.6
684250 2008 RK90	4.28	0.12	3.5	15.5

There are a number of other asteroids in unstable orbits here.

For example, 3552 Don Quixote:

a = 4.27 a.u., e = 0.71, i = 31°

The motion of three resonant groups of asteroids relative to Jupiter



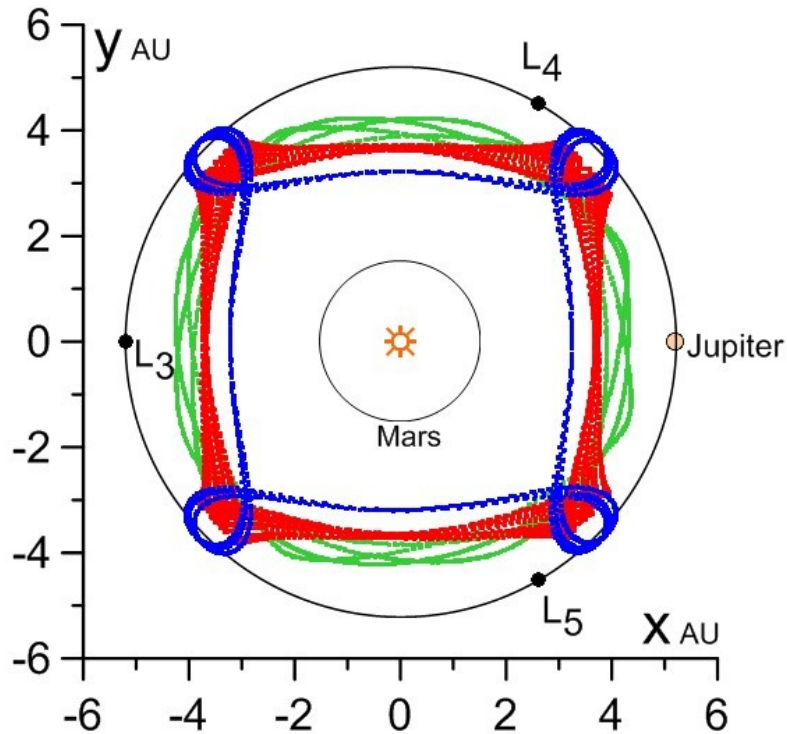
Three resonant regions:

Jupiter Trojans (1:1): coordinates of 8000 asteroids on the Jupiter's orbit plane.

Hildas (3:2): coordinates of 3000 asteroids .

Thule group (4:3): Asteroid (185290) 2006UB219 movement in a rotating coordinate system.

The motion of asteroids in the Thule group



279 Thule

$a=4.27$ au, $e=0.04$, $i=2.33^\circ$

185290 2006 UB219

$a=4.30$ au, $e=0.13$, $i=7.12^\circ$

186024 2001 QG207

$a=4.29$ au, $e=0.25$, $i=3.24^\circ$

loops at the corners of a square occur if $e>0.15$

Secular perturbations

1. Classical secular perturbations
2. Lidov-Kozai mechanism (LKM) or 'von Zeipel-Lidov-Kozai' due to the work by von Zeipel (1910).

Analytical theories assume that no mean-motion resonances take place (\cite{br1950, lid1962, koz1962}). However, there are very strong mean motion resonances in the asteroid belt.

Y. Kozai, 1985 studied LKM in the region of resonances by using a semi-analytical method. It was shown that in resonant regions the LKM manifests itself differently than in non-resonant ones. For the 4:3 resonance it was found: Max e (or Min i) are at $\omega = 0^\circ, 180^\circ$ (not at usual $\omega = 90^\circ, 270^\circ$).

T. Vinogradova, 2024 used another method to study the LKM in Hildas and Trojans. Vinogradova T. A., Lidov-Kozai mechanism in Hildas and Jupiter Trojans, Celestial Mechanics, Vol. 136, 17, 2024. This method is used in the present work.

Method

The method used long-term numerical integration.

An orbit evolution of real asteroids in the Thule group was investigated within the framework of restricted three-body problem.

Elements of the Jupiter's orbit were fixed at the moment corresponding to the epoch of asteroid orbital elements. As a source of initial osculating elements, the IAU MPC database of orbital elements was employed.

1. Long-term numerical integration was performed.
2. Classical secular perturbations were identified.
3. Classical perturbations were excluded from osculating elements. An exclusion of the perturbations can be made very easy using a coordinate transformation formula.
4. After excluding classical perturbations, LKM i - and e - oscillations can become visible in the distributions of elements.

Exclusion of classical secular perturbations

secular theory equations for i :

$$\sin i \cos \Omega = B \cos (-a t + D) + P \quad (1)$$

$$\sin i \sin \Omega = B \sin (-a t + D) + Q$$

for e, ϖ – the same, but (+ a t ...)

coordinate transformation formula:

$$\begin{aligned} \sin i \sin \Omega &= \cos \Omega_0 \sin i_1 \sin \Omega_1 + \sin \Omega_0 \cos i_0 \sin i_1 \cos \Omega_1 + \sin \Omega_0 \sin i_0 \cos i_1 \\ \sin i \cos \Omega &= -\sin \Omega_0 \sin i_1 \sin \Omega_1 + \cos \Omega_0 \cos i_0 \sin i_1 \cos \Omega_1 + \cos \Omega_0 \sin i_0 \cos i_1 \quad (2) \\ \cos i &= -\sin i_0 \sin i_1 \cos \Omega_1 + \cos i_0 \cos i_1 \end{aligned}$$

Elements: i, Ω – osculating, i_1, Ω_1 – proper, Ω_0, i_0 - forced

$$i_0 \ll, i_1 \ll \rightarrow \cos i_0 = 1, \cos i_1 = 1$$

$$\sin i \cos \Omega = \sin i_1 \cos (\Omega_0 + \Omega_1) + \sin i_0 \sin \Omega_0 \quad (3)$$

$$\sin i \sin \Omega = \sin i_1 \sin (\Omega_0 + \Omega_1) - \sin i_0 \cos \Omega_0$$

coordinate transformation formula agrees with secular theory equations

Classical secular perturbations $i(\Omega)$, $e(\varpi)$

i - variations depends on the longitude of the ascending node $i(\Omega)$

e -variations -- on the longitude of perihelion $e(\varpi)$.

The results of numerical integration in Fig.1, Fig.2 show well classical secular perturbations in the orbital elements of this asteroid.

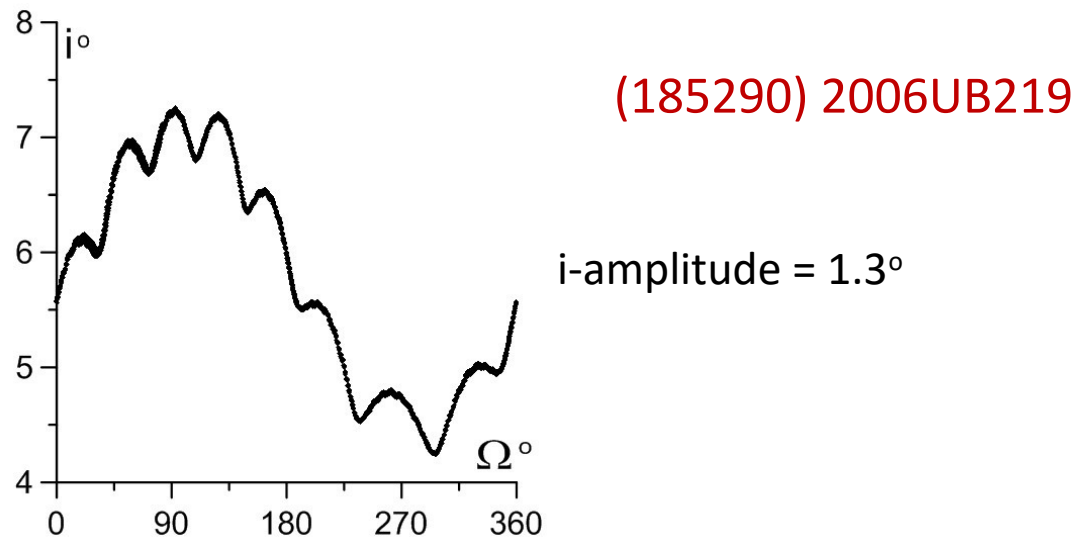


Fig.1 Classical secular perturbations $i(\Omega)$.
Second order oscillations are caused by LKM.
1 Ω -period = 4 ϖ -period = 8 oscillations.

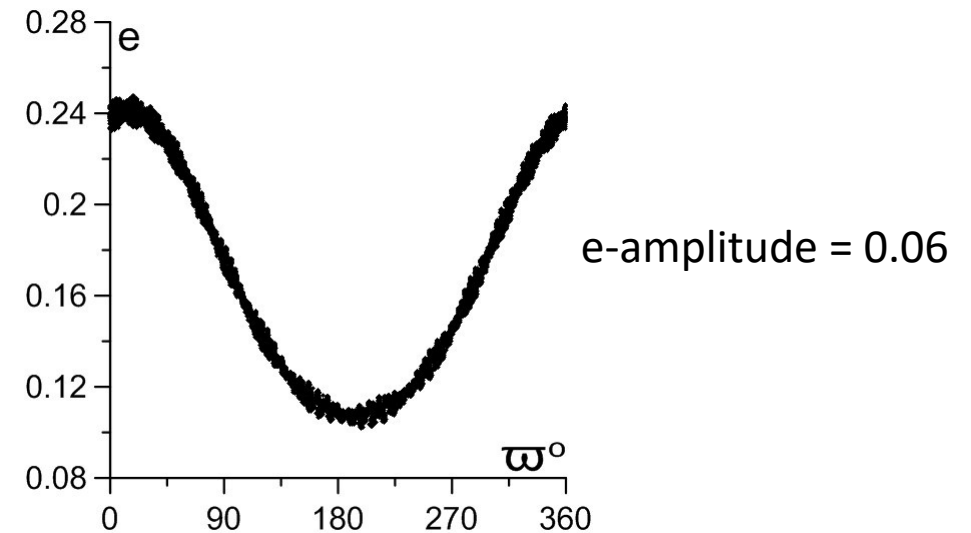


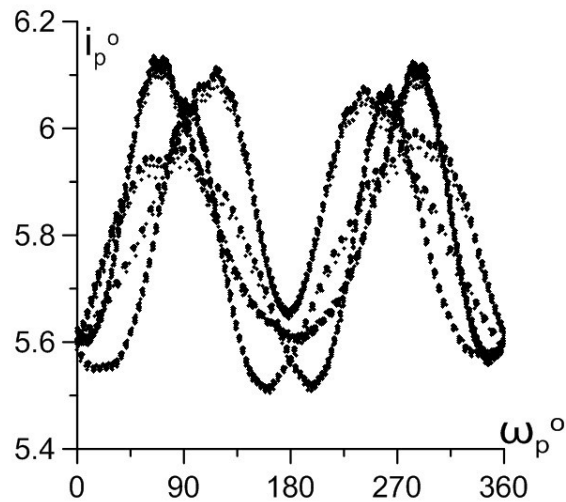
Fig.2 Classical secular perturbations $e(\varpi)$. LKM oscillations are not visible due to the small amplitude.
 ϖ retrograde motion.

Lidov-Kozai mechanism $i(\omega)$, $e(\omega)$ in the Thule group

The LKM is a secular perturbation which causes long-period oscillations in the inclination i and eccentricity e depending on a perihelion argument ω . Under the influence of LKM i and e change in concert due to $(1 - e^2) \cos^2(i) = \text{const}$.

After excluding classical perturbations, LKM i , e -oscillations can become visible in distributions of proper elements (i_p, ω_p) and (e_p, ω_p) .

Max e (or Min i) are at $\omega = 0^\circ, 180^\circ$ (not at usual $\omega = 90^\circ, 270^\circ$).



(185290) 2006UB219
 ω retrograde motion

Fig.3 LKM secular perturbations $i(\omega)$.
LKM i -oscillations amplitude = 0.2°

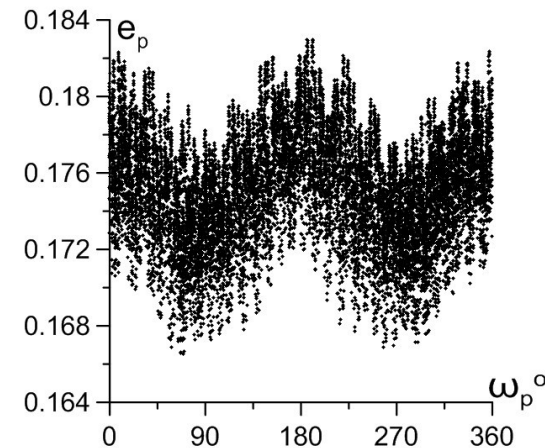
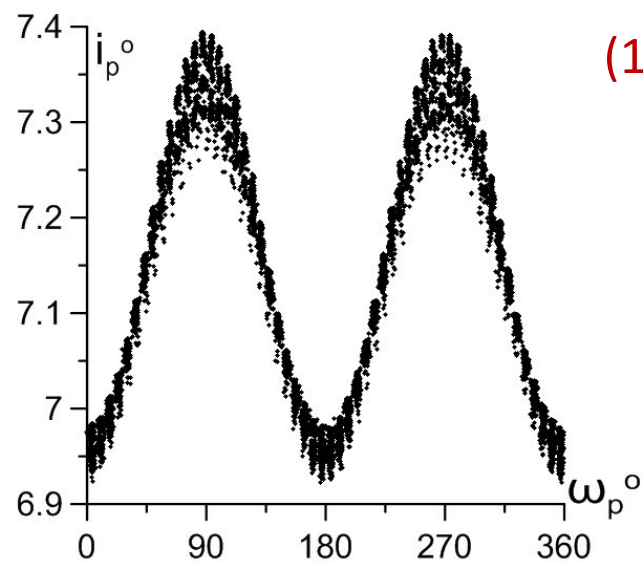


Fig.4 LKM secular perturbations $e(\omega)$.
LKM e -oscillations amplitude = 0.004

Lidov-Kozai mechanism in the Thule group

Jupiter's orbit eccentricity $e_{\text{jup}} = 0$



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Fig.5 LKM $i(\omega)$.

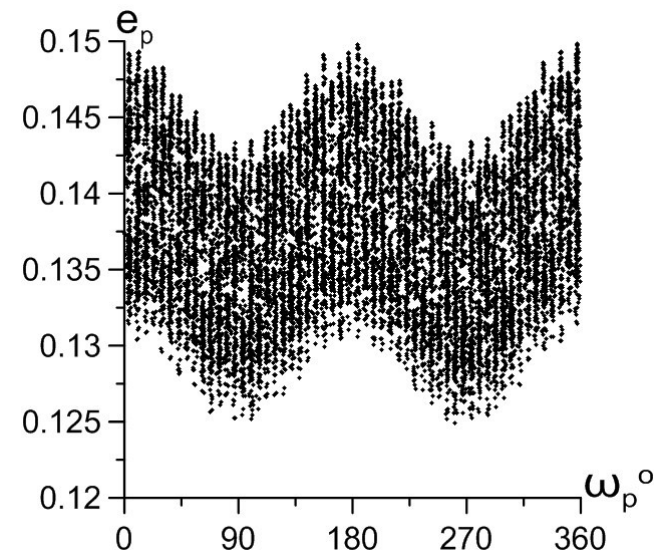


Fig.6 LKM $e(\omega)$.

20898 Fountainhills in resonance Lidov-Kozai ω -libration

$a=4.29$ au, $e=0.47$, $i=44.8^\circ$ $H=11.36^m$

The asteroid's trajectory does not draw a square, but occupies the entire space between the orbits of Mars and Jupiter

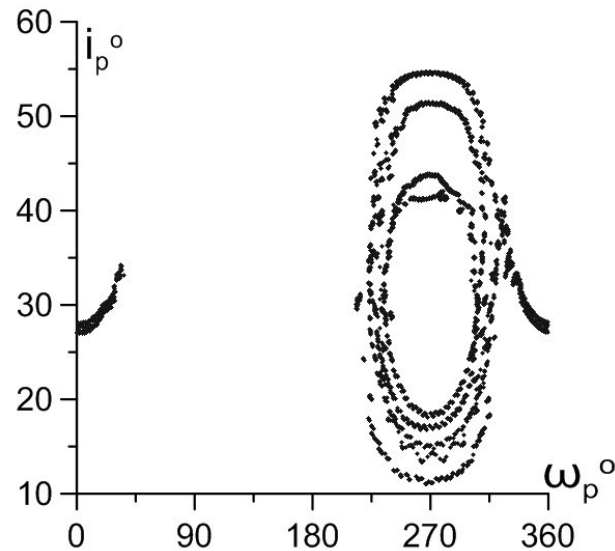


Fig.5 LKM $i(\omega)$. ω -libration

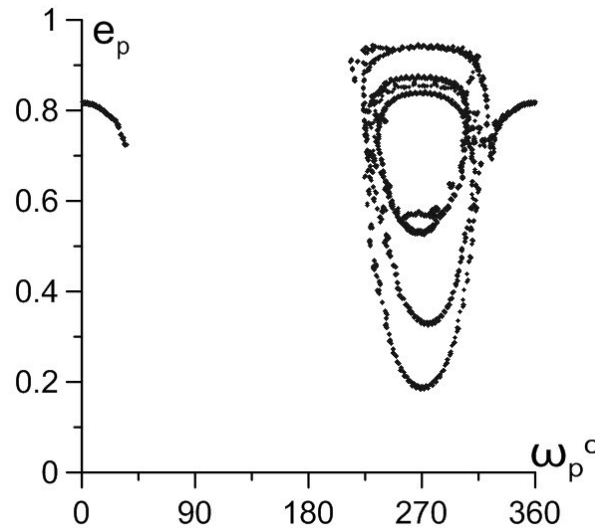


Fig.6 LKM $e(\omega)$.

ω -libration is observed here at $i = 10^\circ$,
whereas usually at $i = 39^\circ$

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Thank you for your attention!