

Velocities-distance optimization of the rotation model for a Galactic ensemble of massive young stars with masers

Effects of parallax reduction

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Spatial-kinematic modeling (SKM) of our Galaxy, the Milky Way, is an analysis of kinematics that allows the kinematic parameters of the centroid of Galactic objects or diffuse matter to depend on the spatial position of the centroid.

Spatial-kinematic modeling. II

In addition to the research issues, spatial-kinematic modeling makes it possible to directly determine a number of fundamental (universal and non-universal) characteristics:

- the distance from the Sun to the center of the Galaxy R_0 ;
- the rotation curve $\theta(R)$, where θ is the linear velocity of rotation;
- the angular and linear velocities of rotation on the solar circle $\theta_0 \equiv \theta(R_0)$ and $\omega_0 \equiv \omega(R_0)$, respectively;
- parameters of the residual motion of the Sun, $(u_{\odot}, v_{\odot}, w_{\odot}) = \mathbf{V}_{\odot}$;
- the angular and linear velocities of the Sun's rotation around the center of the Galaxy, $\theta_{\odot}$ and $\omega_{\odot}$, respectively;
- other derived characteristics (e.g., the circular linear velocity of the Galaxy at R_0 , θ_c).

Spatial-kinematic modeling. III

The very possibility to solve such problems arises due to the allocation of some kind of **subsystem** of the Galaxy. The allocation of various subsystems is based on the astrophysical nature of objects/matter, which...

- determines the very possibility of obtaining for each object the observational data necessary for conducting spatial and kinematic modeling —
 - coordinates, i.e., Galactic longitude l and latitude b ;
 - velocity characteristics, i.e., heliocentric velocity V_r , proper motion in longitude μ_l and latitude μ_b (directly μ_α , μ_δ);
 - heliocentric distance r ;
- gives reason to expect that the selected subsystem is **homogeneous** in terms of spatial and kinematic properties. Otherwise, we will get «the average temperature in the hospital», which does not represent any of the subsystems.

Assuming a **cylindrical model of rotation of centroids** of objects around the center of the Galaxy, the model values of the velocity components of the object with coordinates l , b and r are as follows.

$$\begin{aligned} V_{r,\text{mod}} &= V_{r,\text{rot}} + V_{r,\odot}, \\ V_{r,\text{rot}} &= [\omega(R) - \omega_0] R_0 \sin l \cos b, \\ V_{r,\odot} &= -u_{\odot} \cos l \cos b - v_{\odot} \sin l \cos b - w_{\odot} \sin b; \end{aligned} \quad (1)$$

$$\begin{aligned} k\mu'_{l,\text{mod}} &= k\mu'_{l,\text{rot}} + k\mu'_{l,\odot}, \\ k\mu'_{l,\text{rot}} &= [\omega(R) - \omega_0] \left(\frac{R_0 \cos l}{r} - \cos b \right) - \omega_0 \cos b, \\ k\mu'_{l,\odot} &= (u_{\odot} \sin l - v_{\odot} \cos l)/r; \end{aligned} \quad (2)$$

$$\begin{aligned} k\mu_{b,\text{mod}} &= k\mu_{b,\text{rot}} + k\mu_{b,\odot}, \\ k\mu_{b,\text{rot}} &= -[\omega(R) - \omega_0] \frac{R_0}{r} \sin l \sin b, \\ k\mu_{b,\odot} &= (u_{\odot} \cos l \sin b + v_{\odot} \sin l \sin b - w_{\odot} \cos b)/r; \end{aligned} \quad (3)$$

$$\mu'_l \equiv \frac{dl}{dt} \cos b = \mu_l \cos b, \quad \mu_l \equiv \frac{dl}{dt}; \quad \mu_b \equiv \frac{db}{dt};$$

$k = 4.7406$ is the coefficient of conversion μ [mas/yr] in μ [km/s/kpc] for r [kpc];

$$R(r) = \sqrt{R_0^2 + r^2 \cos^2 b - 2R_0 r \cos l \cos b}; \quad (4)$$

$u_\odot$, $v_\odot$, $w_\odot$ are the components of the **residual velocity of the Sun**, i.e., the components of the Sun's motion relative to the Subsystem Standard of Rest in the directions $(l, b) = (0^\circ, 0^\circ)$, $(l, b) = (90^\circ, 0^\circ)$, and $b = 90^\circ$, respectively.

In the case of a **planar** subsystem, the linear rotation velocity of the centroids $\theta = \omega R$ can be considered a function that depends only on the Galactocentric distance R : $\theta = \theta(R)$.

Since the rotation curves of the outer galaxies are flat over a large range of radii, we use a model in the form of a Taylor polynomial to represent precisely $\theta(R)$ [and not $\omega(R)$]:

$$\Theta_n(R) = \sum_{i=0}^n \frac{\theta_i}{i!} (\Delta R)^i, \quad n \geq 1, \quad \theta_i \equiv \left. \frac{d^i \theta}{dR^i} \right|_{R=R_0}, \quad (5)$$

$$\Delta R \equiv R - R_0.$$

$$\begin{aligned} (\omega - \omega_0) &= \left(\frac{\theta}{R} - \omega_0 \right) \approx \\ &\approx \left[\theta_0 + \theta_1 \Delta R + \frac{1}{2} \theta_2 (\Delta R)^2 + \dots + \frac{1}{n!} \theta_n (\Delta R)^n - \omega_0 R \right] R^{-1}, \end{aligned} \quad (6)$$

$$\theta_0 - \omega_0 R = \omega_0 R_0 - \omega_0 R = -\omega_0 \Delta R,$$

$$(\omega - \omega_0) \approx \left[(\theta_1 - \omega_0) \Delta R + \sum_{i=2}^n \frac{\theta_i}{i!} (\Delta R)^i \right] R^{-1}. \quad (7)$$

Let's introduce the **Oort constant**:

$$A \equiv -\frac{1}{2} R_0 \omega'(R_0) = -\frac{1}{2} R_0 \left. \frac{d(\theta/R)}{dR} \right|_{R=R_0} = -\frac{1}{2} (\theta_1 - \omega_0). \quad (8)$$

Then

$$(\omega - \omega_0) \approx \left(-2A \Delta R + \sum_{i=2}^n \frac{\theta_i}{i!} (\Delta R)^i \right) R^{-1}. \quad (9)$$

$$\theta_1 = -2A + \omega_0. \quad (10)$$

$$V_{r,\text{mod}} = V_{r,\text{rot}} + V_{r,\odot},$$

$$V_{r,\text{rot}} = T_n(\Delta R) \frac{R_0}{R} \sin l \cos b, \quad (11)$$

$$V_{r,\odot} = -u_{\odot} \cos l \cos b - v_{\odot} \sin l \cos b - w_{\odot} \sin b;$$

$$k\mu'_{l,\text{mod}} = k\mu'_{l,\text{rot}} + k\mu'_{l,\odot},$$

$$k\mu'_{l,\text{rot}} = T_n(\Delta R) \left(\frac{R_0 \cos l}{r} - \cos b \right) R^{-1} - \omega_0 \cos b, \quad (12)$$

$$k\mu'_{l,\odot} = (u_{\odot} \sin l - v_{\odot} \cos l)/r;$$

$$k\mu_{b,\text{mod}} = k\mu_{b,\text{rot}} + k\mu_{b,\odot},$$

$$k\mu_{b,\text{rot}} = T_n(\Delta R) \frac{R_0}{Rr} \sin l \sin b, \quad (13)$$

$$k\mu_{b,\odot} = (u_{\odot} \cos l \sin b + v_{\odot} \sin l \sin b - w_{\odot} \cos b)/r;$$

$$T_n(\Delta R) \equiv -2A\Delta R + \sum_{i=2}^n \frac{\theta_i}{i!} (\Delta R)^i, \quad \Delta R \equiv R - R_0, \quad (14)$$

$$R(r, R_0) = \sqrt{R_0^2 + r^2 \cos^2 b - 2R_0 r \cos l \cos b.}$$

Conventional fitting: The least squares method. I

1D fitting:

$$\min \sum_{j=1}^N \left\{ \frac{[V_{r,j} - V_{r,\text{mod}}]^2}{(\sigma_{V_r})_j^2} \right\}, \quad (15)$$

N is the sample size.

3D fitting:

$$\min \sum_{j=1}^N \left\{ \frac{[V_{r,j} - V_{r,\text{mod}}]^2}{(\sigma_{V_r})_j^2} + \frac{[\mu'_{l,j} - \mu'_{l,\text{mod}}]^2}{(\sigma_{\mu'_l})_j^2} + \frac{[\mu_{b,j} - \mu_{b,\text{mod}}]^2}{(\sigma_{\mu_b})_j^2} \right\}. \quad (16)$$

But $V_{r,\text{mod}} = V_{r,\text{mod}}(\mathbf{r})$, $\mu'_{l,\text{mod}} = \mu'_{l,\text{mod}}(\mathbf{r})$, $\mu_{b,\text{mod}} = \mu_{b,\text{mod}}(\mathbf{r})$

⇒ Optimizations (15) and (16) are tasks with **internal noise**

⇒ **Systematic bias** of the results.

Conventional fitting: The least squares method. II

This problem became especially relevant after the appearance of mass joint determinations of proper motions and trigonometric parallaxes—e.g., catalogs of maser sources (Reid et al. 2019; VERA Collaboration, Hirota et al. 2020), the Gaia catalog (Gaia Collaboration, Vallenari et al. 2023)

Distant characteristic.

- Distance modulus (relative, i.e., photometric, distances):
 $d = m - M, r [\text{kpc}] = 10^{0.2d-2}.$
- Trigonometric parallax (absolute distances):
 $r [\text{kpc}] = 1/\varpi [\text{mas}].$

A common problem of spatial-kinematic modeling:

reference objects must be distant →

random errors of reference distances are also large:

$$\sigma_r = \frac{\ln 10}{5} \sigma_d r, \quad \sigma_r = \sigma_\varpi r^2, \quad r_{\text{crit}} = \frac{\ln 10}{5} \frac{\sigma_d [\text{mag}]}{\sigma_\varpi [\text{mas}]}.$$
 (17)

4D fitting. I

The method was proposed in N.+ (2024), N. (2025).

Let the distances r be determined by the trigonometric parallax method: $r = 1/\varpi$, where ϖ — the measured parallax value with the average measurement error σ_ϖ . Then the likelihood function is the product of four probability densities for V_r , μ'_l , μ_b , and ϖ :

$$L = \prod_{j=1}^N \frac{1}{\sqrt{2\pi}(\sigma_{V_r})_j} \exp \left\{ \frac{[V_{r,j} - V_{r,\text{mod}}(\varpi_{0,j})]^2}{2(\sigma_{V_r})_j^2} \right\} \times \\ \times \frac{1}{\sqrt{2\pi}(\sigma_{\mu'_l})_j} \exp \left\{ \frac{[\mu'_{l,j} - \mu'_{l,\text{mod}}(\varpi_{0,j})]^2}{2(\sigma_{\mu'_l})_j^2} \right\} \times \\ \times \frac{1}{\sqrt{2\pi}(\sigma_{\mu_b})_j} \exp \left\{ \frac{[\mu_{b,j} - \mu_{b,\text{mod}}(\varpi_{0,j})]^2}{2(\sigma_{\mu_b})_j^2} \right\} \frac{1}{\sqrt{2\pi}\sigma_{\varpi,j}} \exp \left\{ \frac{(\varpi_j - \varpi_{0,j})^2}{2\sigma_{\varpi,j}^2} \right\}, \quad (18)$$

where $V_{r,j}$, $\mu'_{l,j}$, $\mu_{b,j}$ and ϖ_j are the catalog (measured) values of V_r , μ'_l , μ_b and ϖ for the j -th object;

$\varpi_{0,j}$ is the **reduced parallax** value of the j -th object;

$\Pi_{0,j} \sim N(\varpi_j, \sigma_{\varpi,j}^2)$.

4D fitting. II

$$\mathcal{L} \equiv -\ln L = \mathcal{L}^{(0)} + \mathcal{L}^{(1)}(\mathbf{a}),$$

$$\mathcal{L}^{(0)} = 4N \ln \sqrt{2\pi} + \sum_{j=1}^N [\ln \sigma_{\varpi,j}] = \text{const},$$

$$\begin{aligned} \mathcal{L}^{(1)}(\mathbf{a}) = & \sum_{j=1}^N \left\{ \ln[\sigma_{V_r}(\varpi_{0,j})]_j + \ln[\sigma_{\mu'_l}(\varpi_{0,j})]_j + \ln[\sigma_{\mu_b}(\varpi_{0,j})]_j \right\} + \\ & + \frac{1}{2} \sum_{j=1}^N \min_{\varpi_{0,j}} \left\{ \frac{[V_{r,j} - V_{r,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{V_r}(\varpi_{0,j})]_j^2} + \frac{[\mu'_{l,j} - \mu'_{l,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{\mu'_l}(\varpi_{0,j})]_j^2} + \right. \\ & \left. + \frac{[\mu_{b,j} - \mu_{b,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{\mu_b}(\varpi_{0,j})]_j^2} + \frac{(\varpi_j - \varpi_{0,j})^2}{\sigma_{\varpi,j}^2} \right\}. \end{aligned} \quad (19)$$

4D fitting:

$$\mathcal{L}^{(1)}(\mathbf{a}) \rightarrow \min, \quad (20)$$

$\mathbf{a} = (R_0, \omega_0, A, \theta_2, \dots, \theta_n, u_{\odot}, v_{\odot}, w_{\odot}, \sigma_R, \sigma_{\theta}, \sigma_Z)$ is the vector of model parameters.

The triple of values $(l_j, b_j, \varpi_{0,j})$ defines the point of the (non-orthogonal) projection of the object onto the kinematic model (rotation curve), taking into account all uncertainties.

The variances for velocity characteristics take into account the **natural** (dynamic) velocity dispersion, represented as an ellipsoid of velocities $(\sigma_R, \sigma_\theta, \sigma_Z)$:

$$\begin{aligned}(\sigma_{V_r})_j^2 &= (\tilde{\sigma}_{V_r})_j^2 + (\sigma_{V_r}^*)_j^2, \\(\sigma_{\mu'_l})_j^2 &= (\tilde{\sigma}_{\mu'_l})_j^2 + (\sigma_{\mu'_l}^*)_j^2, \\(\sigma_{\mu_b})_j^2 &= (\tilde{\sigma}_{\mu_b})_j^2 + (\sigma_{\mu_b}^*)_j^2,\end{aligned}\tag{21}$$

where $\tilde{\sigma}_j^2$ are the **measuring** variance;

$\sigma_j^{*2} = \sigma_j^{*2}(l_j, b_j, r | \sigma_R, \sigma_\theta, \sigma_Z, R_0)$ are contributions of **natural dispersion**:

$$\begin{aligned}
 \sigma_{V_r}^{*2} &= \sigma_R^2 \cos^2 \varphi \cos^2 b + \sigma_\theta^2 \sin^2 \varphi \cos^2 b + \sigma_Z^2 \sin^2 b, \\
 \sigma_{V_l}^{*2} &= \sigma_R^2 \sin^2 \varphi + \sigma_\theta^2 \cos^2 \varphi, \\
 \sigma_{V_b}^{*2} &= \sigma_R^2 \cos^2 \varphi \sin^2 b + \sigma_\theta^2 \sin^2 \varphi \sin^2 b + \sigma_Z^2 \cos^2 b,
 \end{aligned} \tag{22}$$

$$\sigma_{\mu_l'}^* = \frac{\sigma_{V_l}^*}{k r}, \quad \sigma_{\mu_b}^* = \frac{\sigma_{V_b}^*}{k r}. \tag{23}$$

Here, φ is an angle with a vertex at the point of projection of the object on the Galactic plane between the direction to the center of the Galaxy and the line $l = \text{const}$, $b = 0^\circ$; the angle φ is counted counterclockwise when viewed from the North Pole of the Galaxy:

$$\sin \varphi = \frac{R_0 \sin l}{R(r)}, \tag{24}$$

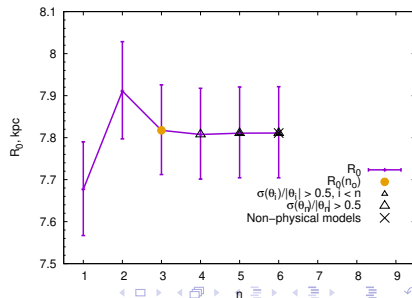
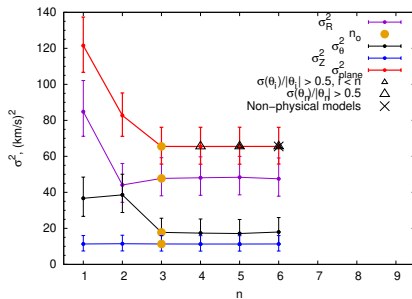
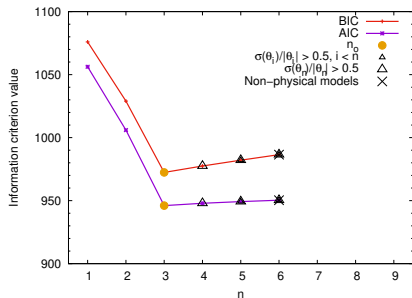
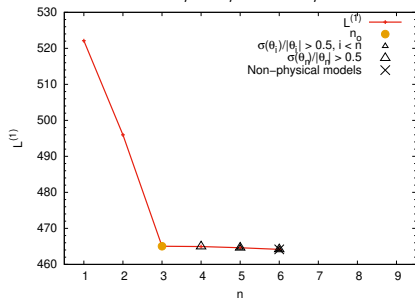
$$\cos \varphi = \frac{R_0 \cos l - r \cos b}{R(r)}. \tag{25}$$

Algorithm. I

- Level I. The solution for $n = \text{const}$ and a fixed sample of N objects.
- Level II. Optimization of the order of the model rotation curve (N., 1999a,b; N., 2025). The optimal order, n_o , model was selected based on the dependence of the characteristics that reflect the quality of the solution on the order of the model:
 - 1 $\mathcal{L}^{(1)}(n)$;
 - 2 $\sigma_{\text{plane}}^2(n)$, $\sigma_{\text{plane}}^2 \equiv \sigma_R^2 + \sigma_\theta^2$;
 - 3 $\text{BIC}(n)$, $\text{AIC}(n)$, where
$$\text{BIC} \equiv 2 \min (\mathcal{L}^{(1)}) + k \ln N \text{ (Bayesian information criterion),}$$
$$\text{AIC} \equiv 2 \min (\mathcal{L}^{(1)}) + 2k \text{ (Akaike information criterion),}$$
$$k \text{ is the number of free parameters;}$$
 - 4 presence of *inaccurate kinematic coefficients* of the approximating polynomial $\Theta_n(R)$: $\sigma_{\theta_i}/|\theta_i| > 0.5 \quad \forall 2 \leq i \leq n$ (special t -criterion for testing the hypothesis $\mathcal{H}_0$: $\theta_i = 0$ for the significance level of 0.05);
 - 5 presence of *fictitious structure* of the $\Theta_n(R)$ model at the edges of the R interval covered by the data (*non-physicality* of the model).

Algorithm. II

HMSFRs: All, 4D, $L' = 1$, $N = 198$



- Level III. Exclusion of outliers.

For results obtained in a fixed optimal order n_o , objects with redundant residuals are searched and excluded.

We use a flexible algorithm for excluding objects with large residuals.

- One-dimensional algorithm (N., 2012).
- Generalization of one-dimensional algorithm to a four-dimensional field of residuals based on statistics χ_4^2 .
 - 1 For each of these algorithms, the limiting value κ of the corresponding criterion statistic is determined.
 - 2 Then the number L of sample objects for which the statistic exceeds κ is found.
 - 3 If $L > 1$, then $L - L'$ objects with the largest residual modules are excluded from further consideration, where $L' \geq 1$ is a parameter of this algorithm.

$$P(L) = \frac{e^{-1}}{L!}. \quad (26)$$

Тогда

$$\begin{aligned} P(L \geq 2) &\cong 0.264 = \mathcal{P}(L' = 1), \\ P(L \geq 3) &\cong 0.080, \\ P(L \geq 4) &\cong 0.019 = \mathcal{P}(L' = 3), \\ P(L \geq 5) &\cong 0.0031. \end{aligned} \quad (27)$$

where $\mathcal{P}(L') \equiv P(L \geq L' + 1)$ is the probability of a “false alarm”.

Based on these values, the number $L = 4$ can be considered clearly redundant. Therefore, to estimate natural variances, we should take $L' = 3$. Then fix the obtained (unbiased) variances and solve the problem with respect to the remaining non-dispersive parameters, but with $L' = 1$.

- [Level IV. [Exclusion of isolated objects.](#)]

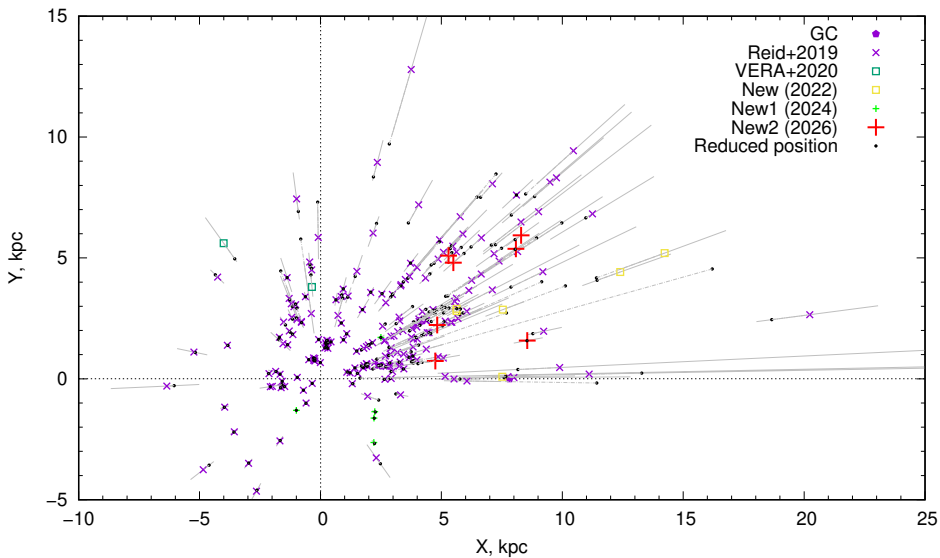
Exclusion of objects at the edges of the R interval covered by data, which are isolated from the main part of the sample objects by their position on the rotation curve after parallax reduction.

A single application of the algorithms of levels I–IV is one iteration. If, according to its results, the sample has decreased, in the next iteration there is a new solution to the problem for the remaining sample, i.e., algorithms of levels I and II are executed. Then the algorithms of level III, IV are applied again. The shutdown occurs when exceptions are no longer needed in the next iteration

Since about 2007, a number of groups have been intensively engaged in measuring trigonometric parallaxes and the corresponding movements of maser sources: interferometry with an ultra-long base of masers is carried out at a methanol frequency (CH_3OH) 12 GHz, H_2O 22 GHz, $\text{SiO} \rightarrow$ [masers with trigonometric parallaxes](#) to distances of ~ 10 kpc with 3D velocities.

The [masers in the high-mass star-forming regions \(HMSFR\)](#) with extremely young ($\ll 1$ Myr) stars, which may yet have reached the main sequence, are of the greatest interest for studying the kinematics of the Galaxy as a very “cold” subsystem of a thin disk with precise absolute distances even for large heliocentric distances (N. & Veselova, 2018b). These masers represent the most homogeneous subsystem of masers with trigonometric parallaxes.

[This paper](#). New measurements have been added to the database described in N. (2025) for 7 HMSFRs, including 3 new objects (Bian+, 2024; Kumar+ 2025). The final catalog of HMSFRs used here contains $N = 213$ objects.



4D results. I

Final samples: $L' = 3$, $N = 202$; $L' = 1$: $N = 198$;
 $n_o = 3$.

n	R_0 (kpc)	ω_0 $\left(\frac{\text{km/s}}{\text{kpc}}\right)$	A $\left(\frac{\text{km/s}}{\text{kpc}}\right)$	$u_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	$v_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	$w_\odot$ $\left(\frac{\text{km}}{\text{s}}\right)$	σ_R $\left(\frac{\text{km}}{\text{s}}\right)$	σ_θ $\left(\frac{\text{km}}{\text{s}}\right)$	σ_z $\left(\frac{\text{km}}{\text{s}}\right)$	θ_2 $\left(\frac{\text{km/s}}{\text{kpc}^2}\right)$	θ_3 $\left(\frac{\text{km/s}}{\text{kpc}^3}\right)$
3	7.817 $+0.109$ -0.105	28.46 ± 0.21	15.55 $+0.23$ -0.22	6.03 ± 0.83	15.52 ± 0.89	8.21 ± 0.51	6.91 $+0.79$ -0.74	4.22 $+0.85$ -0.86	3.36 $+0.63$ -0.65	-1.29 ± 0.20	1.06 ± 0.14

Derived parameters:

$$\theta_0 = 222.4 \pm 3.7 \text{ km/s},$$

$$\theta_1 = -2.64 \pm 0.54 \text{ km/s},$$

$$\theta_\odot = 238.0 \pm 3.8 \text{ km/s},$$

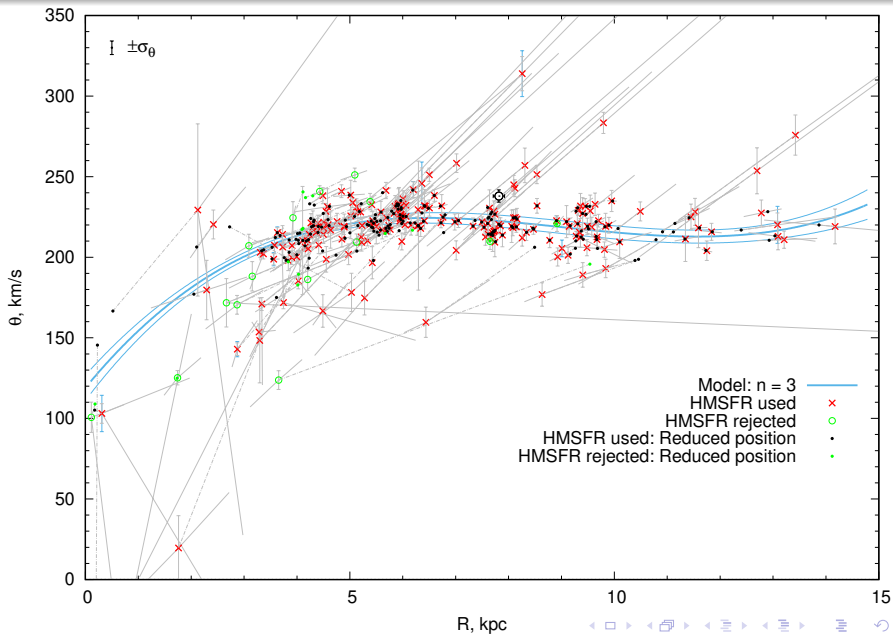
$$\omega_\odot \equiv \omega_0 + v_\odot/R_0 = 30.44 \pm 0.18 \text{ km/s/kpc};$$

$$\theta_c = 233.24 \pm 3.8 \text{ km/s (at azimuth peculiar velocity)}$$

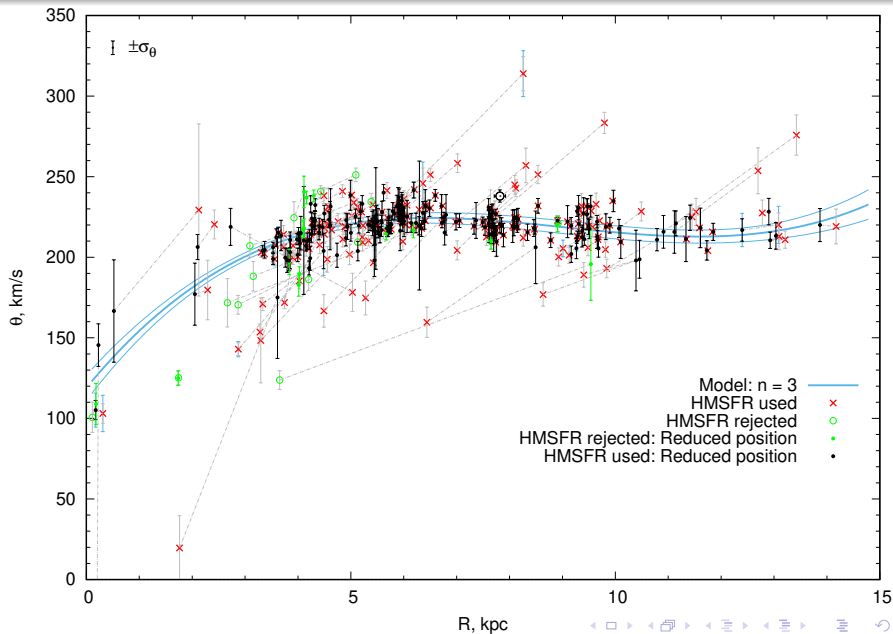
$$v_\odot^{\text{LSR}} = 4.76 \pm 0.49 \text{ km/s, see Ding+, 2019).}$$

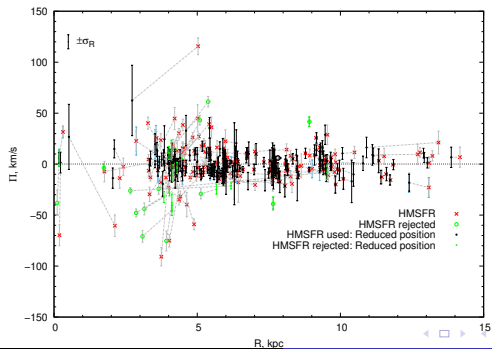
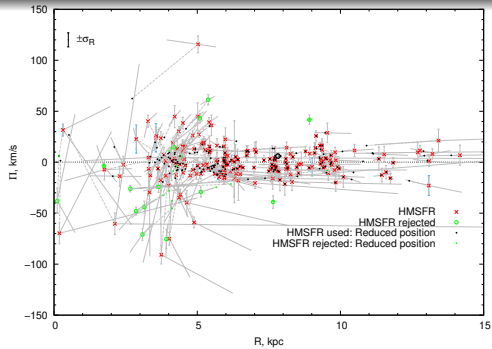
Reid+ (2019): $N = 147$ from ≈ 200 HMSFR masers,
 $R_0 = 8.15 \pm 0.15 \text{ kpc (3D-fitting)}.$

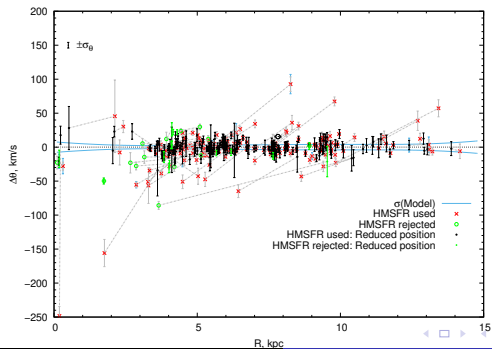
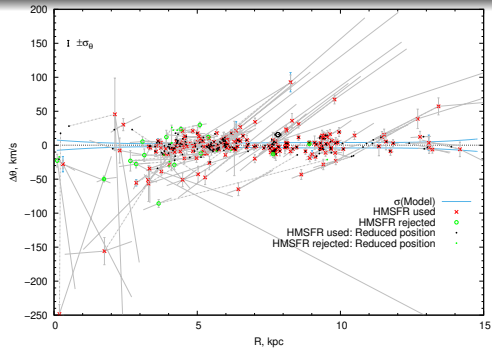
4D results. II

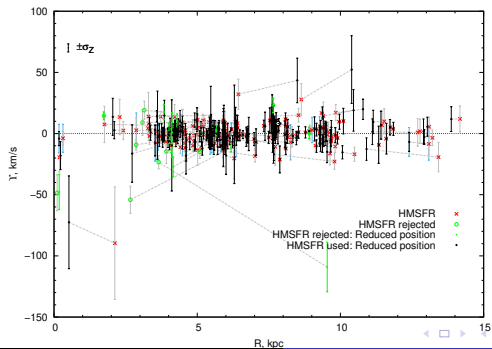
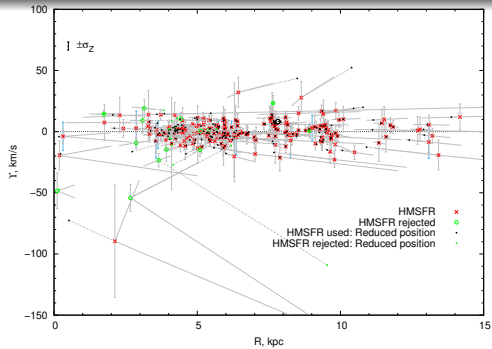


4D results. III









Effects of parallax reduction. I

The more accurate 4D method is much more time-consuming and requires much longer calculations than the 3D method.

What are **the advantages of the 4D method** in terms of its impact on the results, and the systematic errors that can occur when using the simpler 3D approach?

To clarify this issue, **the results obtained by 4D and 3D methods were compared** for...

- the same stages of the algorithms application,
- the same samples (to check the effect of the sample),
- popular restrictions on the sample:
 - on the Galactoaial distance R ,
 - on the relative error of the parallax,
- the variant of the spherical ellipsoid of velocities with fixed components ($\sigma_R = \sigma_\theta = \sigma_Z = 5 \text{ km/s}$) used by Reid (2019).

Some of the significant correlation coefficients are:

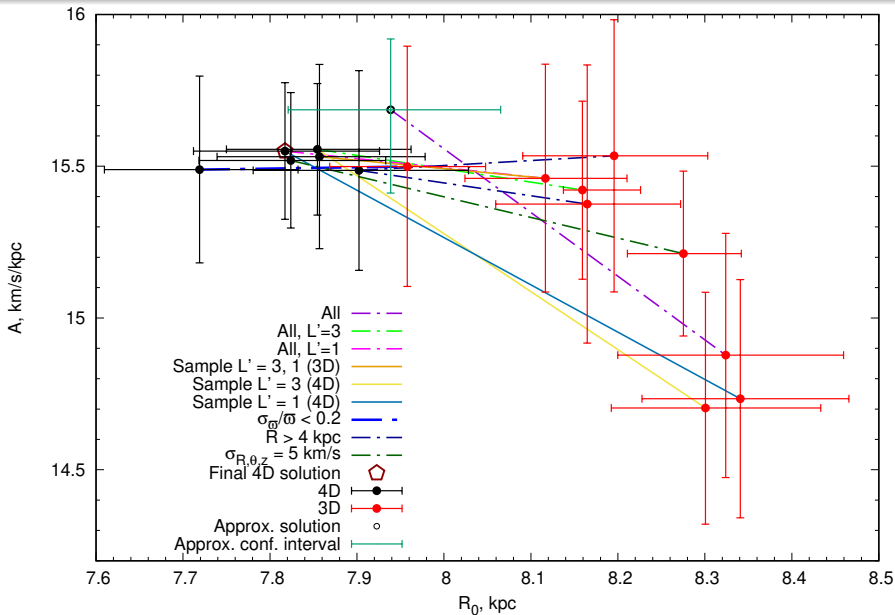
- $\kappa(R_0, A) = -0.33,$
- $\kappa(R_0, v_\odot) = +0.25,$
- $\kappa(u_\odot, A) = +0.22.$

Failure to account for the uncertainty of parallaxes should lead to an **overestimation of the velocity dispersion**.

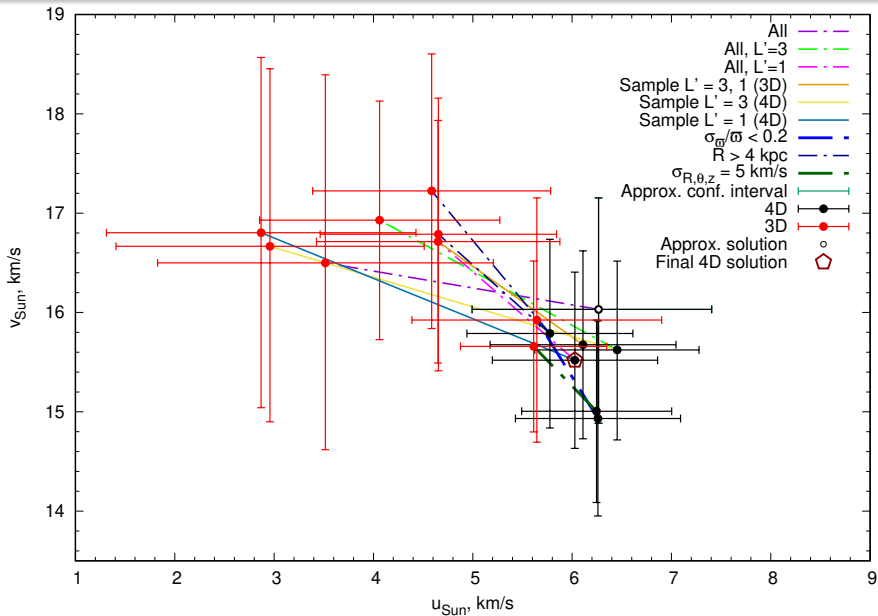
$\implies$

It should be expected that the most systematic effects will appear in the (R_0, A) , $(u_\odot, v_\odot)$, and $(\sigma_R, \sigma_\theta)$ planes.

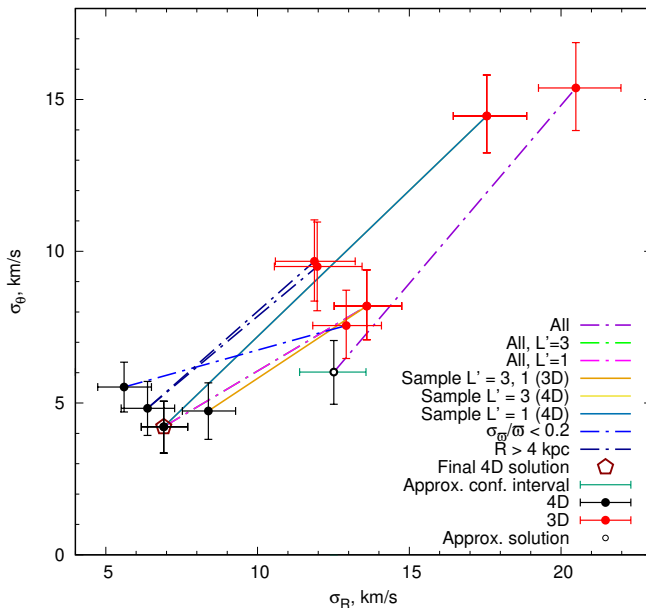
Effects of parallax reduction. III



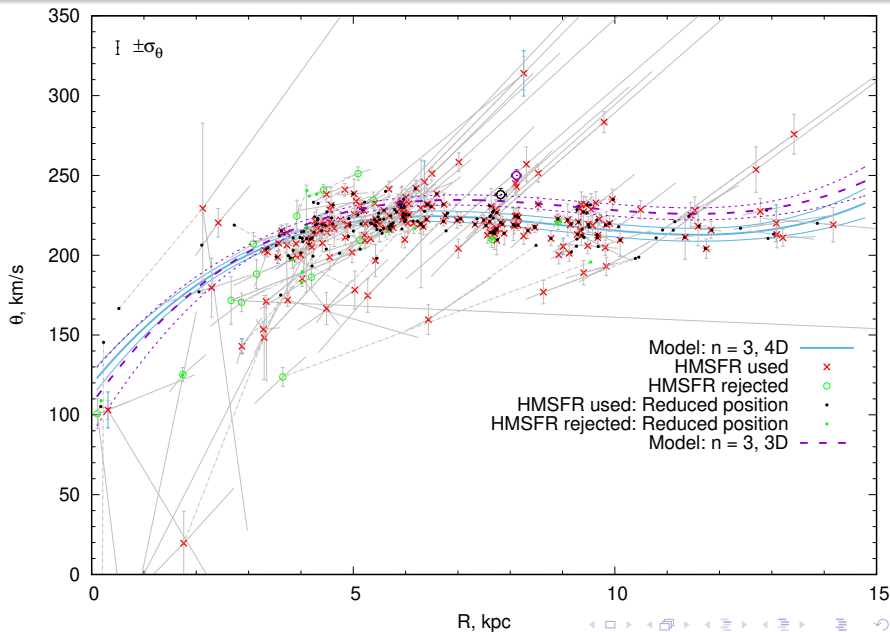
Effects of parallax reduction. IV



Effects of parallax reduction. V



Effects of parallax reduction. VI



The most pronounced **systematic biases** due to the neglect of errors in parallaxes in the 3D method:

- for R_0 the bias is $+(0.2 \div 0.5) \pm 0.1$ kpc ($+3 \div 6\%$),
for A the bias is $(-0.8 \div 0.0) \pm (0.2 \div 0.4)$ km/s/kpc;
- the azimuthal residual velocity of the Sun is overestimated, and the radial velocity is underestimated by about one statistical uncertainty;
- the dispersions σ_R and σ_θ are overestimated by ~ 2 times;
- the 4D method reduces (by decreasing the value of R_0) the level of the rotation curve by ~ 15 km/s in the outer region of the Galaxy ($R > 5$ kpc), which leads to a decrease in the mass of the halo, disk, and entire Galaxy, as well as to a decrease in the radial scale of the disk.

Analysis of differences $\Delta\varpi \equiv \varpi - \varpi_0$:

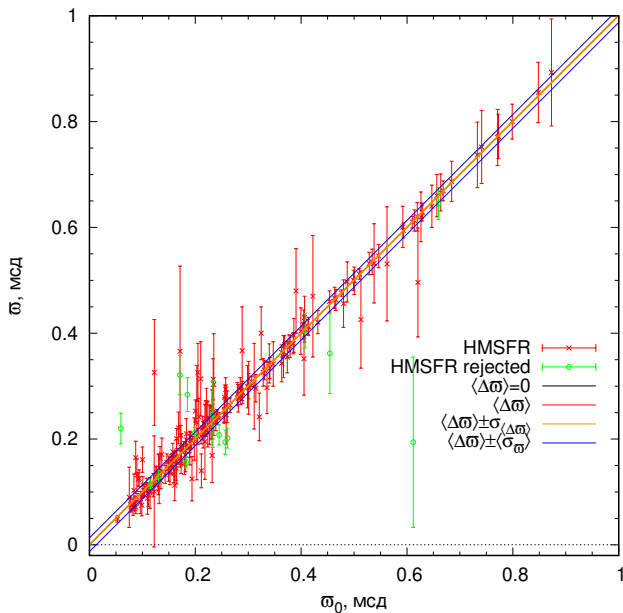
$N = 198$ (all, $L' = 1$),

$\langle \Delta\varpi \rangle = 0.00035 \pm 0.00092$ mas (after equalization)
 ± 0.00114 mas (before equalization),

$\sigma_0 = 0.807 \pm 0.041$,

$\langle \sigma_\varpi \rangle = 0.0129$ mas (after equalization),

$\langle \sigma_\varpi \rangle = 0.0161$ mas (before equalization).



The mechanism of systematic bias

$$V_{r,\text{rot}} = T_n(\Delta R) \frac{R_0}{R} \sin l \cos b, \quad (28)$$

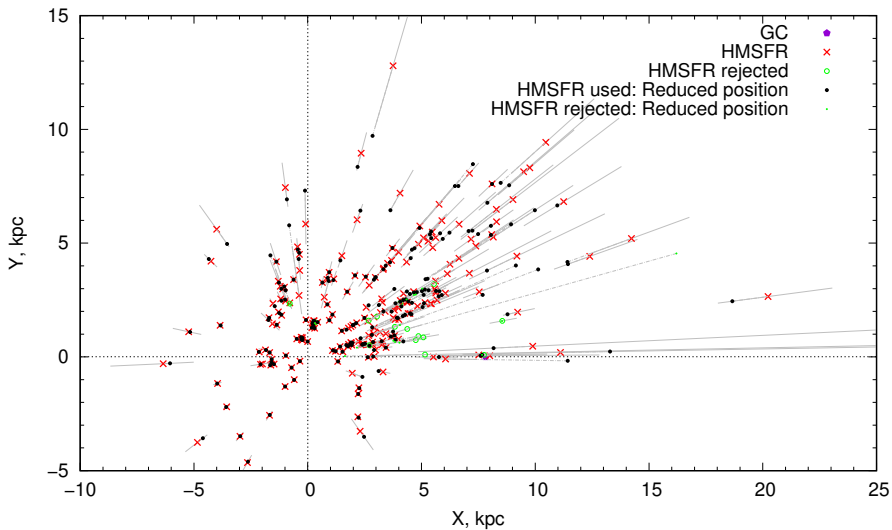
$$T_n(\Delta R) \equiv -2A\Delta R + \sum_{i=2}^n \frac{\theta_i}{i!} (\Delta R)^i, \quad \Delta R \equiv R - R_0, \quad (29)$$

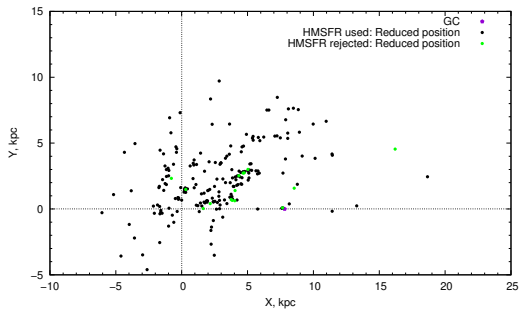
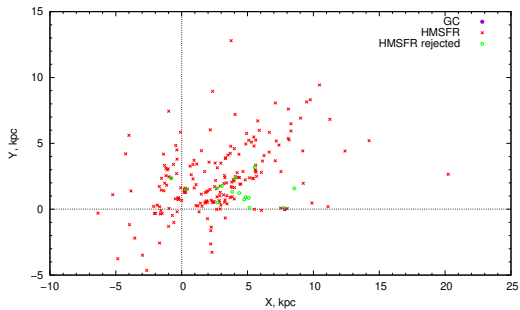
$$R(r, R_0) = \sqrt{R_0^2 + r^2 \cos^2 b - 2R_0 r \cos l \cos b}. \quad (30)$$

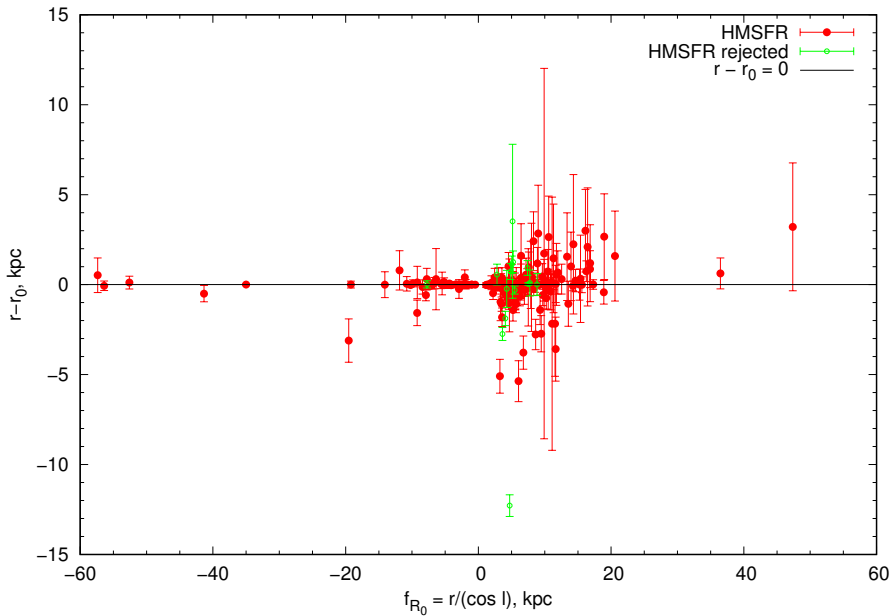
If we decompose R and ΔR in a small neighborhood of the Sun, i.e. at $r \ll R_0$, and restrict ourselves to a first-order term, we get *the first Oort formula*:

$$V_{r,\text{rot}} = Ar \sin 2l \cos^2 b. \quad (31)$$

This means that the estimate of R_0 is determined by **distant** objects (with a large value of $|\sin l|$). Nearby objects have almost no effect on R_0 .

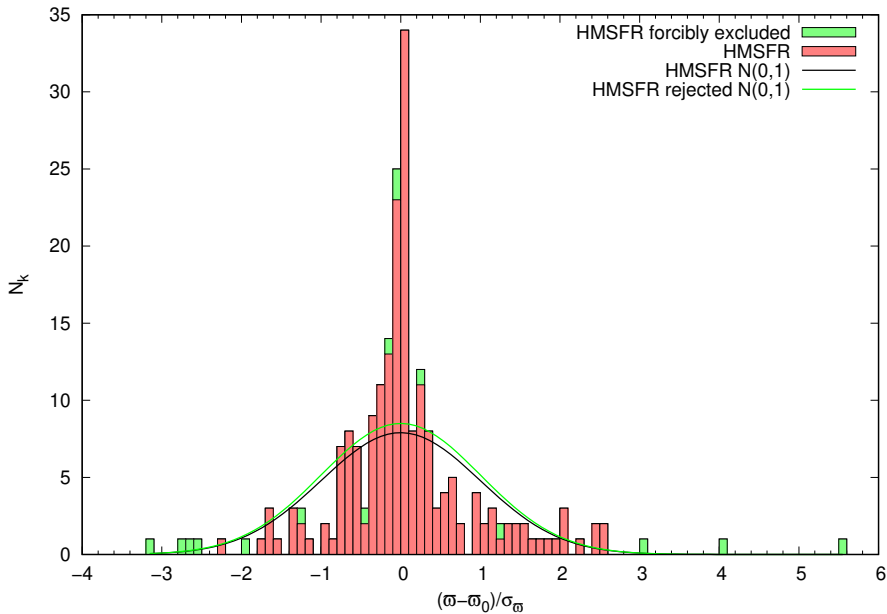


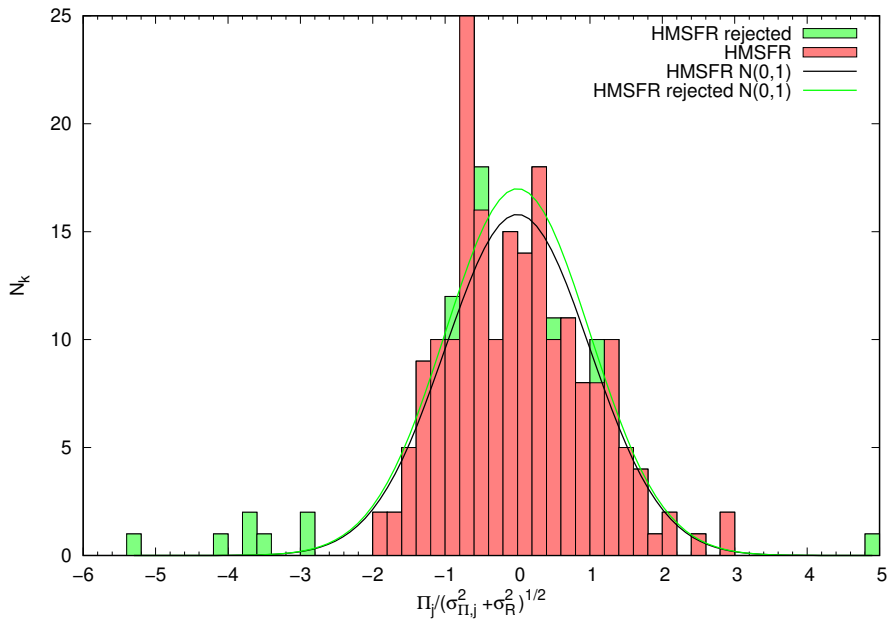


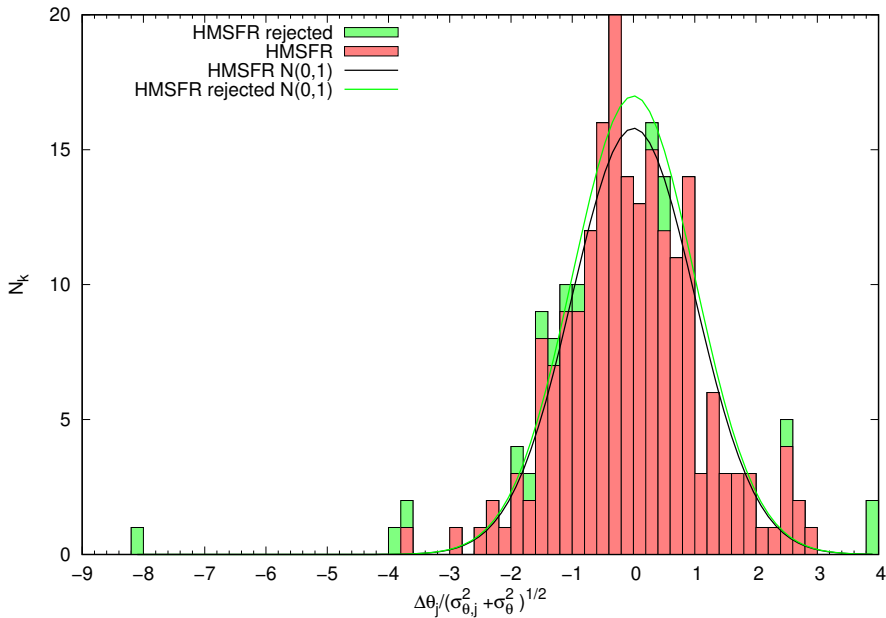


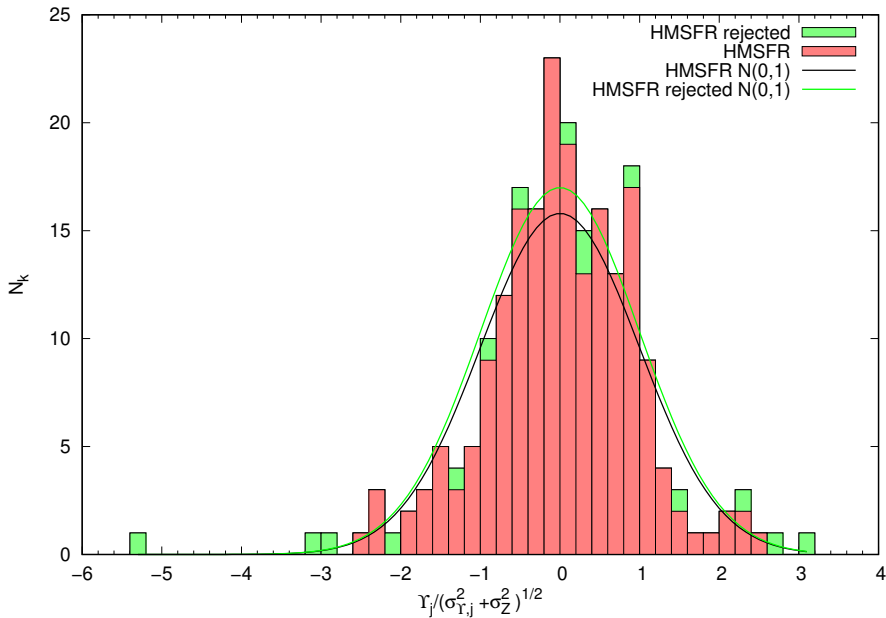
Systematic effects do not occur because of a change in the average distance scale (the 4D method leaves it unchanged), but as a result of the elimination of the nominal expansion relative to the Sun of the distribution of objects in the Galactic plane due to the dispersion of distance estimates («[shrinkflation](#)»): the reduction in the average distances to nominally distant objects reduces the value of R_0 to a greater extent than it increases due to the elongation of close distances.

Such a systematics will occur with [any method of determining distances](#), even in the absence of a systematic error in the distance scale.









- 1 Ignoring parallax errors can lead to significant biases in the results of kinematic modeling of the Galaxy. The complexity of the 4D method is compensated by its systematic accuracy.
- 2 These biases do not occur because of a change in the average distance scale, but as a result of the elimination of the nominal expansion relative to the Sun of the distribution of objects in the Galactic plane due to the dispersion of distance estimates. Such a systematics will occur with any method of determining distances.
- 3 New estimates of the fundamental universal characteristics of the Galaxy, corrected for the specified systematics, have been obtained —
 $R_0 = 7.82 \pm 0.11$ kpc, $\omega_{\odot} = 30.44 \pm 0.18$ km/s/kpc,
 $\theta_{\odot} = 238.0 \pm 3.8$ km/s, and $\theta_c = 233.24 \pm 3.8$ km/s —
as well as the characteristics of the HMSFR maser subsystem.

-  Bian S.B. et al. *On the Structure of the Sagittarius Spiral Arm in the Inner Milky Way*, Astron. J., 2024, 167, 267.
-  Ding P.-J. et al., *Local standard of rest based on Gaia DR2 catalog*, Research in Astron. Astrophys., 2019, 19, 68.
-  Gaia Collaboration, Vallenari A. et al. *Gaia Data Release 3: Summary of the content and survey properties*, Astron. Astroph., 2023, 674, A1
-  Kumar J. et al. *The Expanding 3 kpc Arms Are Neither Expanding nor Spiral Arms but X1 Orbits Driven by the Galactic Bar*, Astroph. J., 2025, 982, 185.
-  Nikiforov, I. I. *Exclusion of measurements with excessive residuals (blunders) in estimating model parameters*, Astron. Astrophys. Trans., 2012, 27, 57.

-  Nikiforov I., Sobolev P., and Veselova A., *4D fitting of the Galactic spatial-kinematic model: masers*, Modern astronomy: from the Early Universe to exoplanets and black holes, RIOR, Moscow, Russia, 2024, pp. 464–470.
-  Nikiforov I., *4D Modeling of Kinematics of a Selected Galaxy Subsystem*, Solar System Research, 2025, 59, 35.
-  Reid M.J. et al., *Trigonometric Parallaxes of High-mass Star-forming Regions: Our View of the Milky Way*, Astrophys. J., 2019, 885, 131.
-  VERA collaboration, Hirota et al. *The First VERA Astrometry Catalog*, Publ. Astron. Soc. Japan., 2020, 72, 50.

Supplement

Актуальные задачи.

- Дальнейшее совершенствование метода.
- Пополнение базы данных о мазерных источниках и изучение их подсистем в Галактике.
- Применение к другим типам опорных объектов (цефеиды и др.).
- Кинематическая калибровка фотометрических шкал расстояний и исправление систематических ошибок измерений параллаксов.
- В перспективе: вклад в решение проблемы природы спиральной структуры Галактики.
- ...

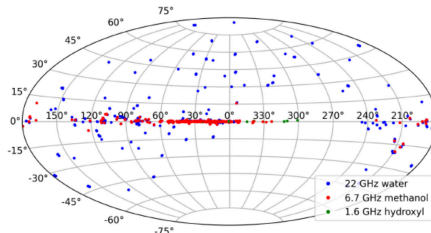
The Maser Monitoring Organisation (M2O)

Communications platform to bring together maser monitoring stations, theorists and follow-up campaigns

17/25

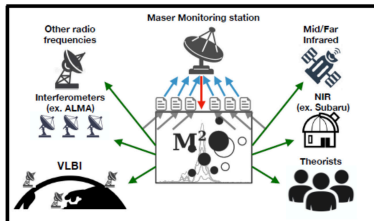


M2O source sky coverage



The fundamental flow of M2O operations:

- 1 New maser flare reported to the M2O
- 2 Disseminate information
 - Consult theorists
 - Check other maser transitions
 - Conduct follow-up observations
- 3 Share results with the team
 - Discuss results from each perspective
 - Plan and author publications
- 4 All publications credit the monitoring station in their author list and acknowledgements



(C) Burns, Ross A. (NAOJ)

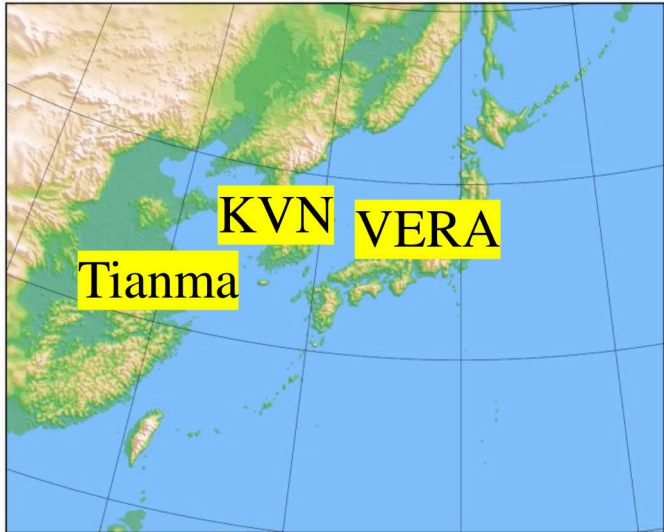
M2O current stats

85 members
14 monitoring stations
>1000 masers monitored
18 flares pursued
23 publications
since 2017 start

M2O website

masermonitoring.org

Meso-scale Analysis domain



Reid+ (2009) attempted to account for the uncertainty of the parallaxes in the framework of the method of least squares, evaluating model parallax by radial velocity, but then, faced with difficulties $\Rightarrow$

Reid+ (2014, 2019) abandoned this approach and returned to the usual “velocity-only” fitting. However, the uncertainty of the distance for many masers turns out to be too large to be ignored.

Rastorguev+ (2017,..) take into account the uncertainty of distances within the framework of the maximum likelihood method (MLM) by including partial derivatives of the first order in distance for velocities in the covariance matrix. However, such accounting ignores the non-Gaussian distribution of errors in distances and is suitable only for small uncertainties of the latter.

A more perfect idea is the [individual reduction of heliocentric distances](#) by including distributions of distant characteristics of objects in the likelihood function.

- [Pont+ \(1994\)](#) applied this approach to photometric distances, using only radial velocities, and while fixing the parameters of the velocity ellipsoid.
- [Aghajani & Lindegren \(2013\)](#); [Ding+ \(2019\)](#): the same approach within the MLM in the case of involving proper motions and parallaxes. However, the analytical solution given by these authors for the reduced (corrected) parallaxes is approximate and can only be applied with small parallax errors.

- Level III. Exclusion of outliers.

For results obtained in a fixed acceptable order $\tilde{n}_o$, objects with redundant residuals are searched and excluded.

We use a flexible algorithm for excluding objects with large residuals.

- One-dimensional algorithm (Nikiforov 2012).

- 1 For a given sample size N , the value of κ is calculated, which satisfies the equation

$$[1 - \psi(\kappa)] N = 1, \quad (32)$$

where $\psi(z)$ is probability integral:

$$\psi(z) = \sqrt{\frac{2}{\pi}} \int_0^z e^{-\frac{1}{2}t^2} dt. \quad (33)$$

Exclusion of outliers. II

- ② We determine the number of objects L of this sample that meet the conditions

$$\frac{|\delta_j|}{\sigma_j} > \kappa. \quad (34)$$

The mathematical expectation of this number is equal to one with a normal distribution of residuals. A larger number of such objects may be considered as redundant with one or another probability.

- ③ If $L > 1$, then $L - L'$ objects with the largest residual modules are excluded from further consideration, where $L' \geq 1$ is a parameter of this algorithm.
- ④ To the remaining objects, i.e., in particular, when $L = 1$, the following criterion is applied: objects with residuals

$$\frac{|\delta_j|}{\sigma_j} > k_\gamma(N), \quad (35)$$

are excluded, where k_γ is the root of the equation

$$1 - [\psi(k_\gamma)]^N = \gamma, \quad (36)$$

here γ is the accepted significance level. We used $\gamma = 0.05$ (approximately the significance level 2σ).

- Generalization of one-dimensional algorithm to a four-dimensional field of residuals based on statistics χ_4^2 .

$$z_j = \frac{[V_{r,j} - V_{r,\text{mod}}(\varpi_{0,j})]^2}{(\sigma_{V_r})_j^2} + \frac{[\mu'_{l,j} - \mu'_{l,\text{mod}}(\varpi_{0,j})]^2}{(\sigma_{\mu'_l})_j^2} + \frac{[\mu_{b,j} - \mu_{b,\text{mod}}(\varpi_{0,j})]^2}{(\sigma_{\mu_b})_j^2} + \frac{(\varpi_j - \varpi_{0,j})^2}{\sigma_{\varpi,j}^2}. \quad (37)$$

$$[1 - F_4(\kappa)]N = 1, \quad (38)$$

$$1 - F_4(z) = P(\chi_4^2 > z) = e^{-z/2} \left(\frac{z}{2} + 1 \right). \quad (39)$$

We determine the number of objects L of this sample that meet the conditions

$$z_j > \kappa. \quad (40)$$

$$P(L) = \frac{e^{-1}}{L!}. \quad (41)$$

Тогда

$$\begin{aligned} P(L \geq 2) &\cong 0.264 = \mathcal{P}(L' = 1), \\ P(L \geq 3) &\cong 0.080, \\ P(L \geq 4) &\cong 0.019 = \mathcal{P}(L' = 3), \\ P(L \geq 5) &\cong 0.0031. \end{aligned} \quad (42)$$

where $\mathcal{P}(L') \equiv P(L \geq L' + 1)$ is the probability of a “false alarm”.

Based on these values, the number $L = 4$ can be considered clearly redundant. Therefore, to estimate natural variances, we should take $L' = 3$. Then fix the obtained (unbiased) variances and solve the problem with respect to the remaining non-dispersive parameters, but with $L' = 1$.

Application of the kinematic method to masers.

Reid+ (2009): $N = 16$, $R_0 = 8.4 \pm 0.6$ kpc.

Reid+ (2014): $N = 80$ out of 103, $R_0 = 8.34 \pm 0.16$ kpc.

Rastorguev+ (2017): $N = 131$, $R_0 \approx 8.24 \pm 0.12$ kpc.

Reid+ (2019): $N = 147$ from ≈ 200 HMSFR masers,
 $R_0 = 8.15 \pm 0.15$ kpc.

VERA Collaboration et al. (2020): $N = 224$, SFRs and RSG masers
from VERA and Reid+ (2019) catalogs,
 $R_0 = 7.92 \pm 0.16_{\text{stat.}} \pm 0.3_{\text{sys.}}$ kpc.

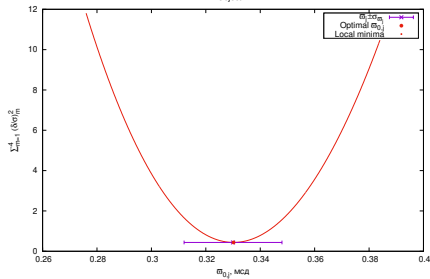
N. (2025). To obtain a sample of HMSFR measurements, we used
the catalogues in Reid+ (2019), VERA+ (2020), supplementing
them with new data from the literature: Xu+ (2021), Sakai+
(2021), Bian+ (2022), Mai+(2023), Hyland+ (2023), Hyland+
(2024). Total data for 280 masers. The full sample of HMSFRs:
 $N = 210$.

$$\varpi_{0,j} : \quad \min \mathcal{L}_{\varpi},$$

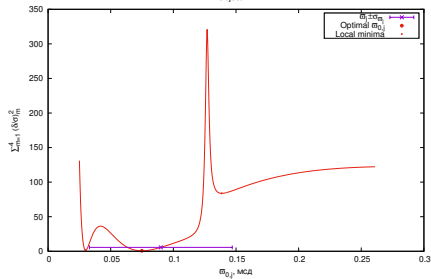
$$\begin{aligned} \mathcal{L}_{\varpi}(\varpi_{0,j}|\mathbf{a}) = & \frac{[V_{r,j} - V_{r,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{V_r}(\varpi_{0,j})]_j^2} + \frac{[\mu'_{l,j} - \mu'_{l,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{\mu'_l}(\varpi_{0,j})]_j^2} + \\ & + \frac{[\mu_{b,j} - \mu_{b,\text{mod}}(\varpi_{0,j})]^2}{[\sigma_{\mu_b}(\varpi_{0,j})]_j^2} + \frac{(\varpi_j - \varpi_{0,j})^2}{\sigma_{\varpi,j}^2}. \end{aligned} \quad (43)$$

Reducing parallaxes. II

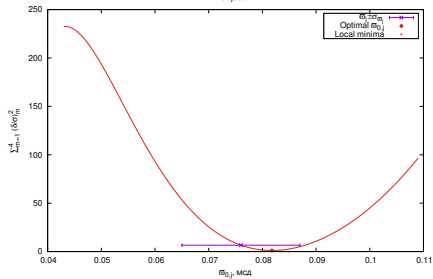
Object 4



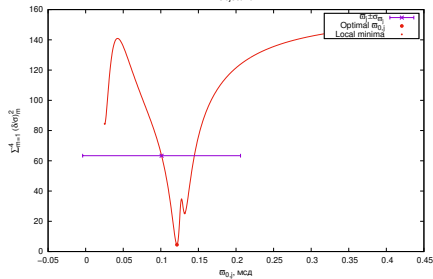
Object 17



Object 7



Object 19



The number n_{ϖ_0} of minima of the $\mathcal{L}_{\varpi}(\varpi_{0,j}|\mathbf{a})$ function
(for the resulting solution at $n_o = 3$, $L' = 1$)

n_{ϖ_0}	$N = 210$ (all)	$N = 197$ ($L' = 1$)
1	184 (87.6%)	174 (88.3%)
2	22 (10.5%)	19 (9.6%)
3	4 (1.9%)	4 (2.0%)

$$\omega_{\odot} \equiv \omega_0 + v_{\odot}/R_0 = 30.40 \pm 0.20 \text{ km/s/kpc}$$

$$\mu_l(\text{Sgr A}^*) = 30.39 \pm 0.03 \text{ km/s/kpc (Xu+, 2022),}$$

$$30.32 \pm 0.34 \text{ km/s/kpc (Gordon+, 2023)} \implies$$

No peculiar velocity of Sgr A* relative to the Galactic
barycenter: $V_l^0(\text{Sgr A}^*) = +0.10 \pm 0.27 \pm 1.6 \text{ km/s,}$
 $+0.63 \pm 2.7 \pm 1.6 \text{ km/s/kpc.}$