

Selecting orbits for safe storage of space debris in the GNSS region

Eduard Kuznetsov,

Tavriz Urazaev, Zhi Li, Quimey Brizuela

Ural Federal University, Yekaterinburg, Russia

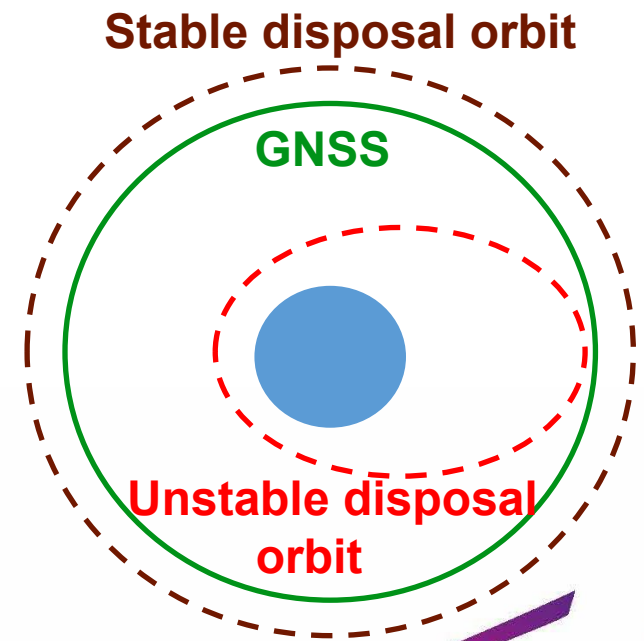
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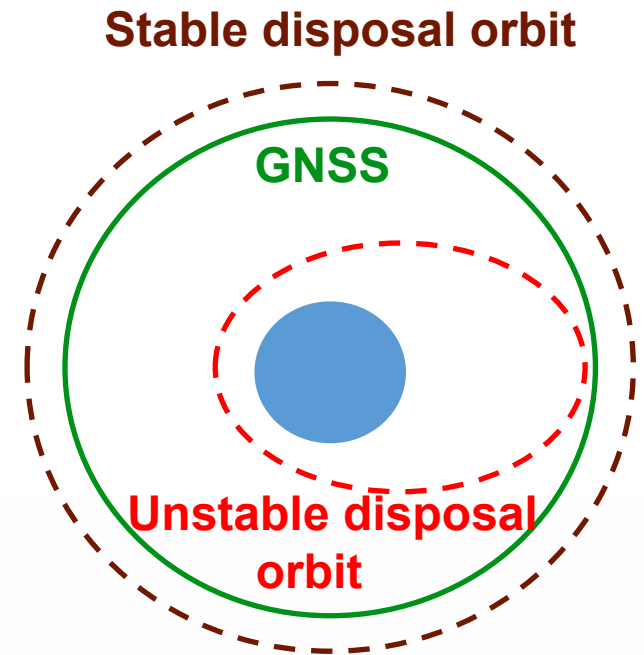
Introduction

- Typically considered two possible disposal strategies for the GNSS region involving either **stable or unstable orbits** from the **eccentricity growth** point of view (see e.g., **Rossi**, CMDA, 100, **2008**; **Deleflie+**, ASR, 47, **2011**; **Pardini & Anselmo**, AcAst, 77, **2012**; **Bordovitsyna+**, SSR, 48, **2014**; ...)
- **Unstable scenarios** seem favorite from point of view the possibility **to dilute the collision risk in GNSS region** and to aim at the re-entry in the atmosphere.
- **Disadvantage**: In recent years, **the risk of collisions in the LEO region has increased significantly**.
- **Stable scenarios** seem **to minimize the interactions** (crossings) with the operational constellations and, therefore, might be preferred for operational reasons.



Aims

- Justification for **the selection of disposal orbits** within the framework of a **Stable scenario**.
- **Requirement** for disposal orbits is that objects in disposal orbits should **not enter the areas of motion** of GNSS and IGSO satellites over tens or hundreds of years (**up to 200 years**).
- During this time, effective systems for **the active debris removal (ADR)** from disposal orbits must be developed.



Methodology

- Our study **is based on** the following fundamental results:
- The theory of the **inclination-dependent-only luni-solar secular resonances** (**Hughes**, Proc. R. Soc. A, **1980**; **Chao**, Adv. Astronaut. Sci., 105, **2000**; **Rossi**, **2007**).
- The theory of the **secondary secular apsidal-nodal resonances due to solar radiation pressure** (**Polyakhova**, Bul. ITA, **1963**; **Polayakhova**, Vest. Len. Uni., **1970**, **1971**, **1972**; **Krivov+**, CMDA, 63, **1996**; **Alessi+**, MNRAS, 473, **2018**).
- Existence of a **stationary point** on the plane “eccentricity – longitude of periapsis” (e, π) (**Chao**, AIAA Paper, AIAA-2006-6514, **2006**; **Lücking+**, AcAst, 77, **2012**).

Secular luni-solar inclination dependent resonances

- The **singly averaged equations** of the third body perturbations (the 6-month and the 14-day terms removed) ([Chao, 2000](#))
- The eccentricity perturbation is

$$\begin{aligned} \frac{de}{dt} = & -\frac{15}{8} e \frac{n_p^2 M_{rat}}{n} \sqrt{1 - e^2} [C1(i, i_p) \sin 2(g - \Delta\Omega) + C2(i, i_p) \sin(2g - \Delta\Omega) \\ & + C3(i, i_p) \sin 2g + C4(i, i_p) \sin(2g + \Delta\Omega) + C5(i, i_p) \sin 2(g + \Delta\Omega)] \end{aligned}$$

M_{rat} is mass ratio ($M_{rat} = 1$ for solar perturbations, $M_{rat} = 182.3$ for lunar perturbations)

$$\Delta\Omega = \Omega - \Omega_p$$

n_p, i_p, Ω_p are mean motion, inclination and longitude of the ascending node of the perturbing body

$C1 - C5$ are coefficients depending only on i and i_p

Secular luni-solar inclination dependent resonances

Resonance term	Critical inclination [deg]	GNSS	Inclination [deg]
$2g - 2\Delta\Omega$	73.1		
$2g - \Delta\Omega$	69.0		
$2g$	63.4	GLONASS	64.8
$2g + \Delta\Omega$	56.0	GPS BeiDou Galileo	55 55 56
$2g + 2\Delta\Omega$	46.4		

- The maximum values of the eccentricity are reached whenever $2g \approx 90^\circ$ or $2g + \Delta\Omega \approx 270^\circ$
- For a spacecraft in the 2:1 resonance, the relation between eccentricity and the angle $(2g + \Omega)$ would be much less smooth due to the interplay between the two different resonances (see Rossi, 2007).

Secondary secular apsidal-nodal resonances due to solar radiation pressure

- **A resonant angle** (Krivov+, CMDA, 63, 1996; Alessi+, MNRAS, 473, 2018)

$$\psi_j = n_1 \Omega + n_2 g + n_3 \lambda_S$$

λ_S is ecliptic longitude of the Sun

If $j = 1$, $\psi_1 = \Omega + g - \lambda_S = \pi - \lambda_S$

- **A stationary point** on the (e, π) plane (Chao, AIAA Paper, AIAA-2006-6514, 2006; Lücking+, AcAst, 77, 2012)

$$e_* = \frac{3}{2} k P \frac{A}{m} \frac{(\cos \epsilon / 2)^2}{a n n_S} \approx 0.01 k \frac{A}{m},$$

- k is the solar radiation pressure coefficient
- $P = 4.56 \times 10^{-6} \text{ N m}^{-2}$
- A/m is the area-to-mass ratio

j	n_1	n_2	n_3
1	1	1	-1
2	1	-1	-1
3	0	1	-1
4	0	1	1
5	1	1	1
6	1	-1	1

$$\pi_* = \lambda_S \quad \text{or} \quad \psi_1 = 0$$

ϵ is the obliquity of the ecliptic to the equator

a is the semi-major axis

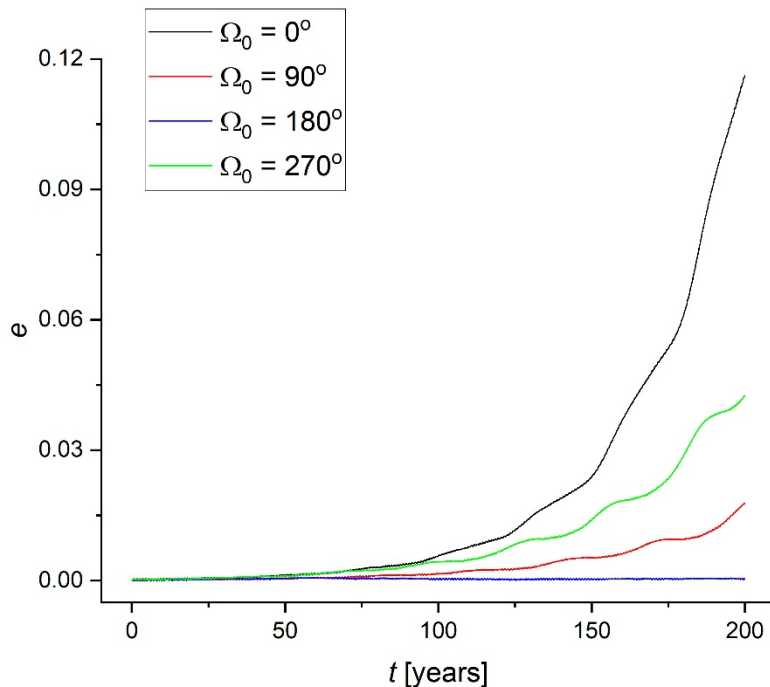
n, n_S are the mean motions of the satellite and the Sun

Numerical simulation

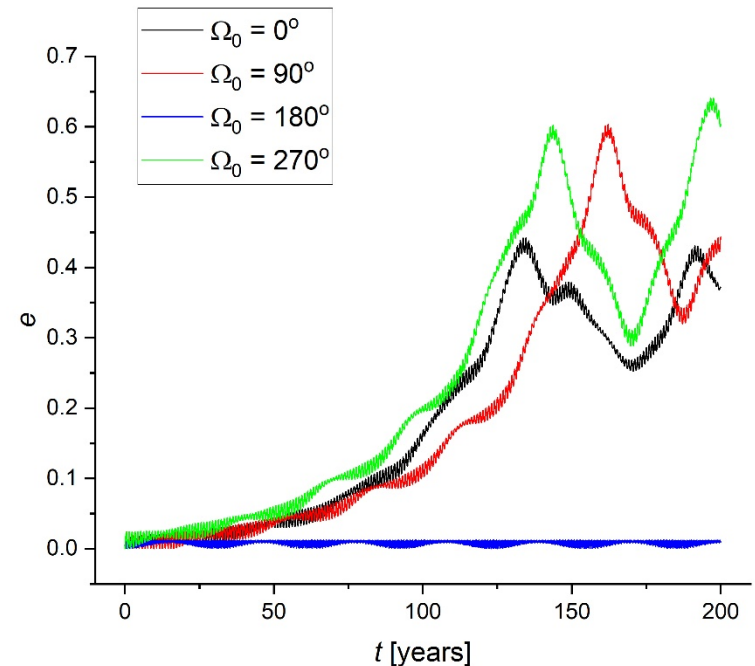
- **Numerical Model of the Motion of an Artificial Satellites** (developed at the Tomsk State University)
- The model of perturbing forces:
 - the gravitational field of the Earth (EGM96, 29 x 29),
 - the gravitation of the Moon and the Sun,
 - the tides of the Earth,
 - the direct solar radiation pressure (the radiation pressure coefficient $k = 1.44$) including the eclipses due to the Earth's shadow,
 - the Poynting-Robertson effect,
 - the atmospheric drag.
- The equations of motion are integrated by Everhart's method of the 19th order.
- Initial epoch $T_0 = 00^h 00^m 00^s 21.03.1998$.
- The integration interval is **200 years**.
- Area-to-mass ratio $A/m = 0.001, 0.01, 0.1, 1 \text{ m}^2 \text{ kg}^{-1}$.

Evolution of the orbital eccentricity e for the **GLONASS** region

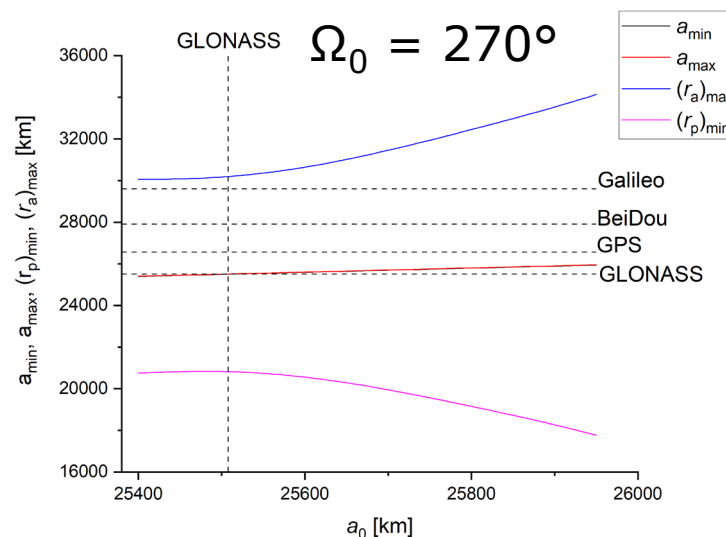
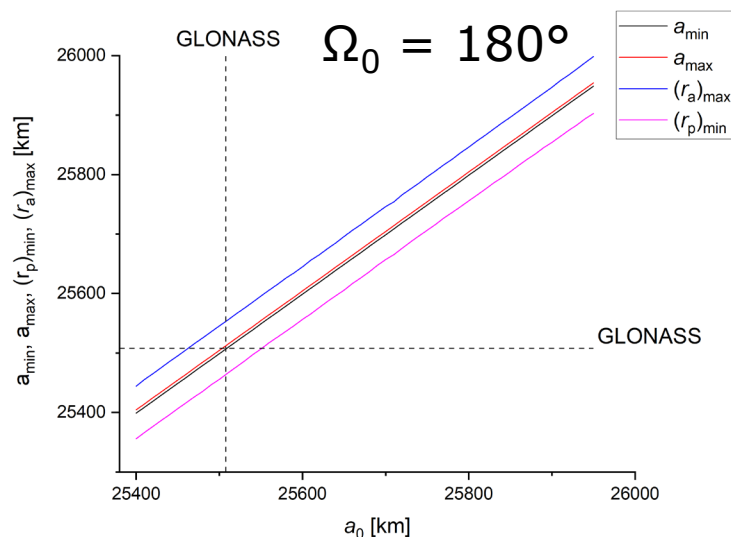
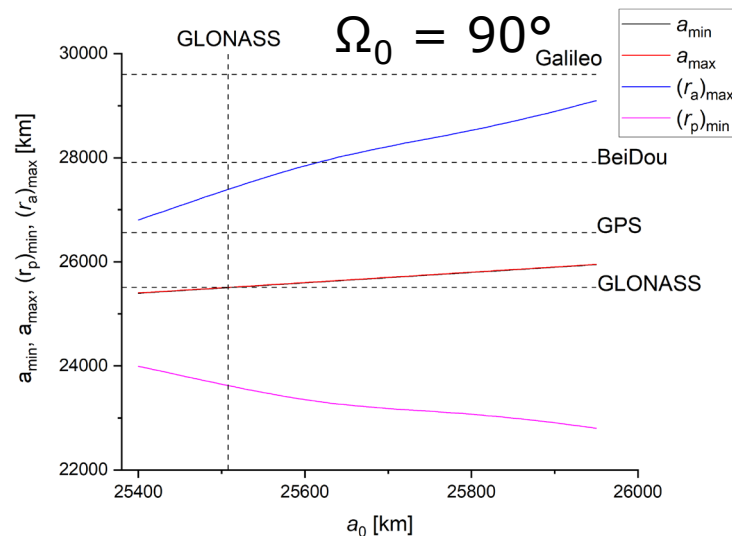
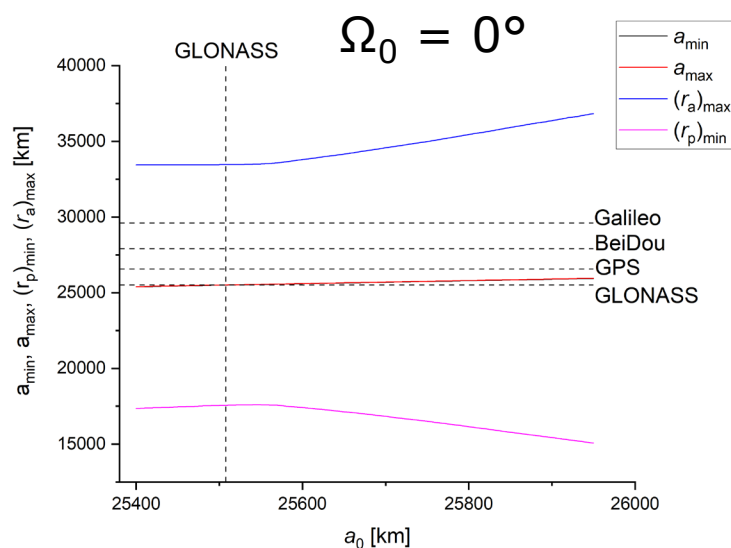
$A/m = 0.01 \text{ m}^2 \text{ kg}^{-1}$



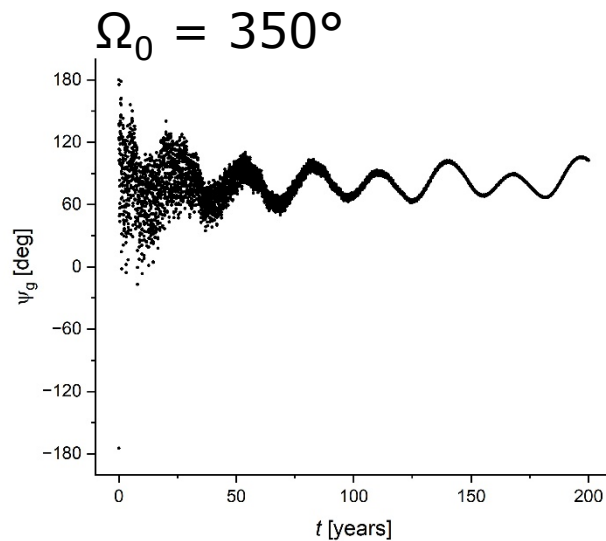
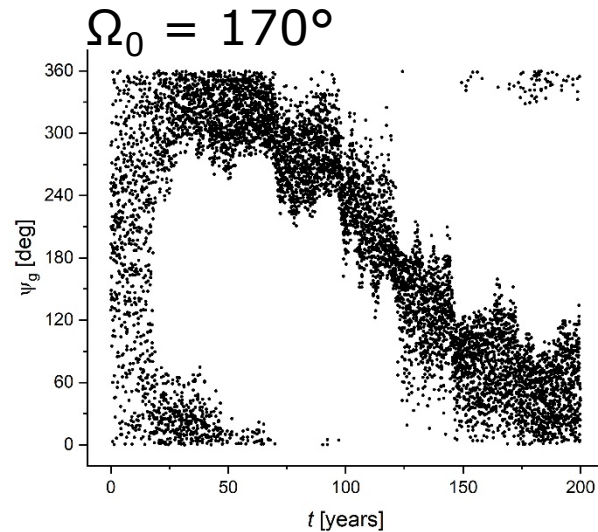
$A/m = 1 \text{ m}^2 \text{ kg}^{-1}$



Evolution of the semi-major axis a , distances in perigee r_p and apogee r_a for **GLONASS** region with $A/m = 0.1 \text{ m}^2 \text{ kg}^{-1}$

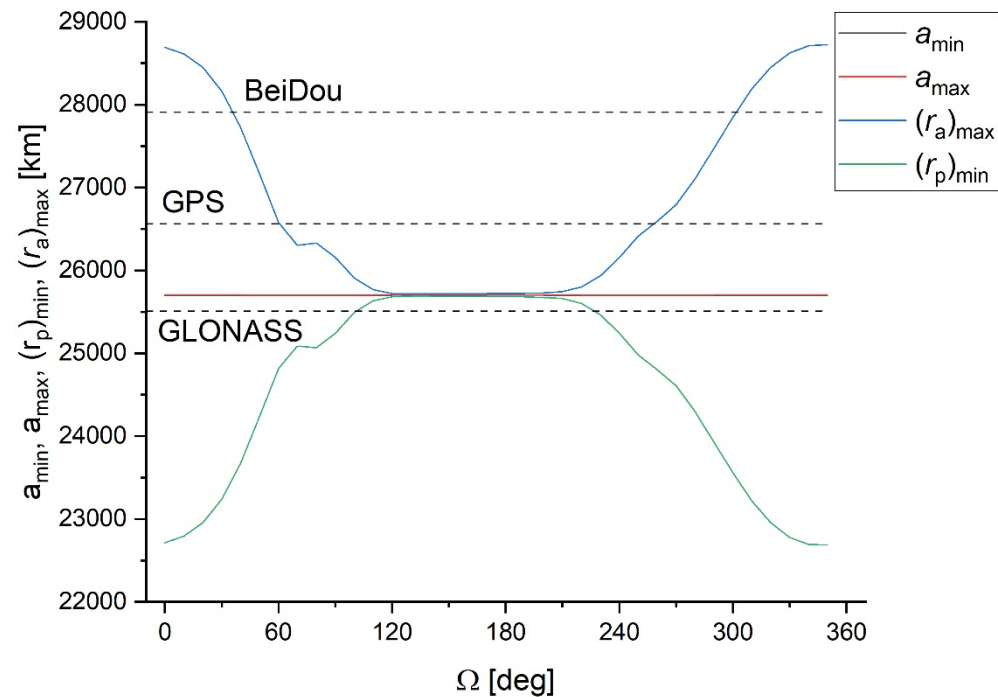


Evolution of $a_{\min}$, $a_{\max}$, $(r_p)_{\min}$, $(r_a)_{\max}$ and resonant angle ψ_g for $a_0 = 25700$ km (GLONASS) with $A/m = 0.01$ m² kg⁻¹



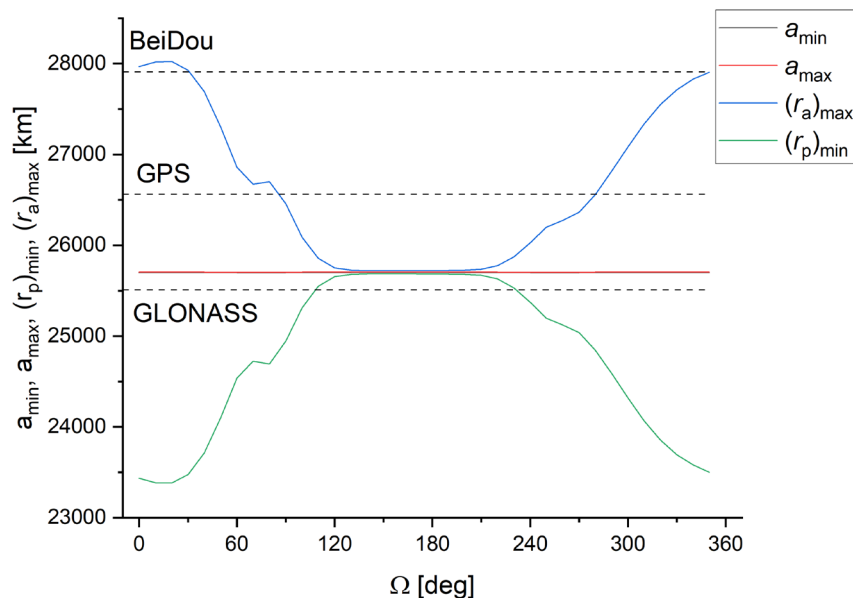
Secular luni-solar resonance

- Resonant angle
 $\psi_g = 2g$

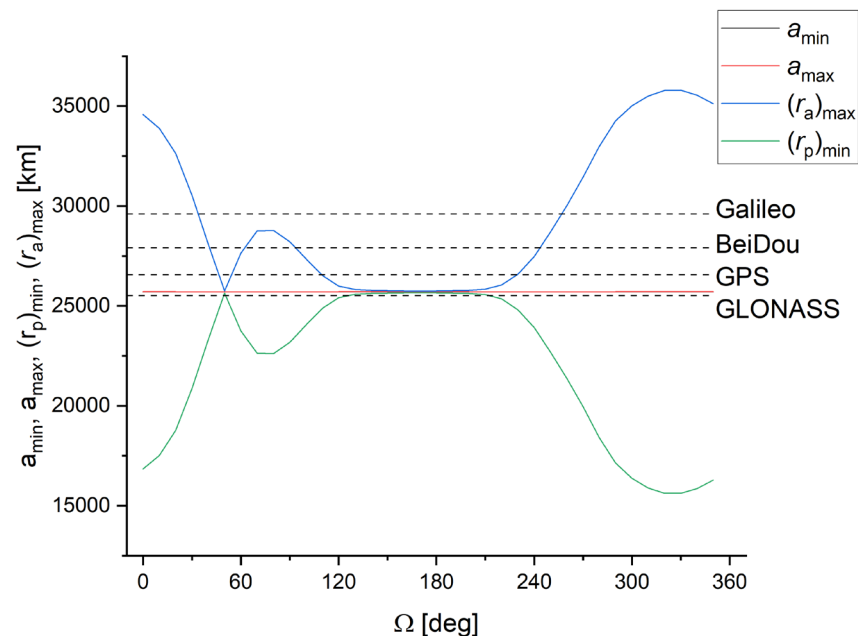


Evolution of the semi-major axis a , distances in perigee r_p and apogee r_a for **GLONASS** region with $a_0 = 25700$ km

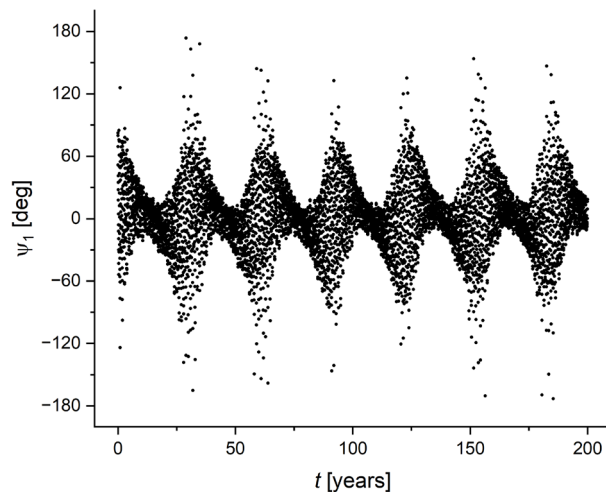
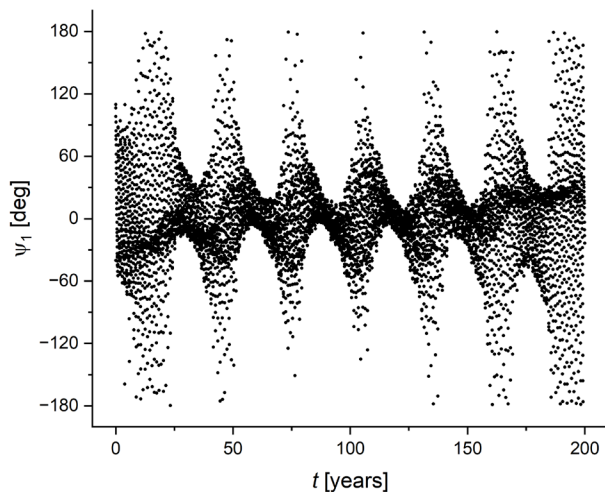
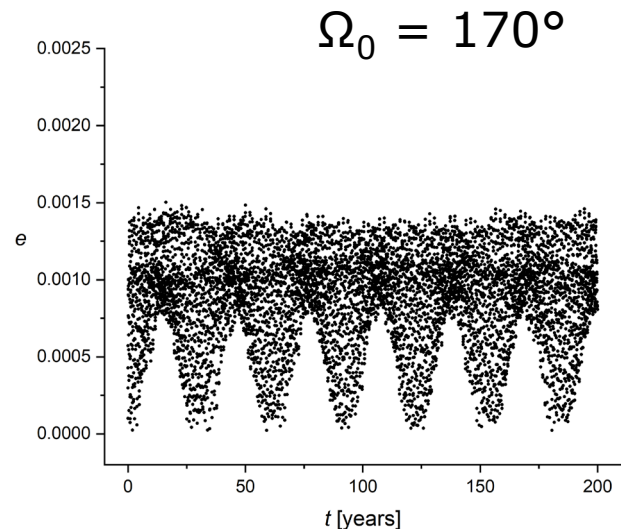
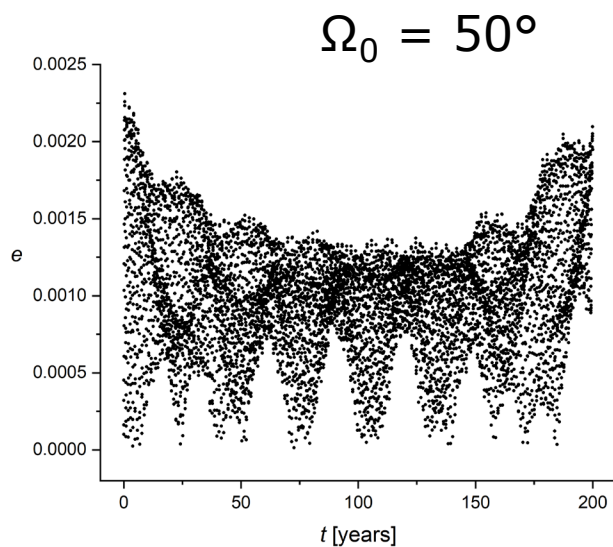
$$A/m = 0.001 \text{ m}^2 \text{ kg}^{-1}$$



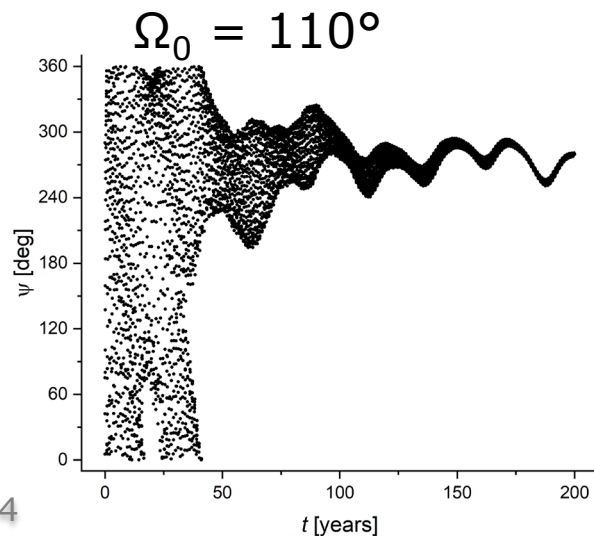
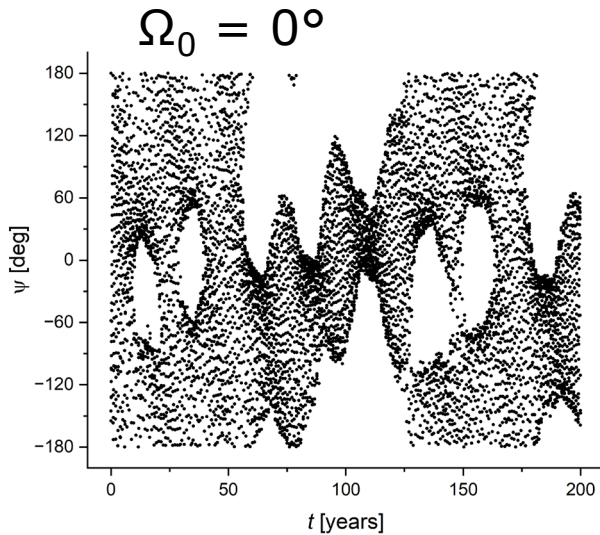
$$A/m = 0.1 \text{ m}^2 \text{ kg}^{-1}$$



Evolution of the eccentricity e and resonant angle $\psi_1 = \pi - \lambda_s$ for $a_0 = 25700$ km (GLONASS) with $A/m = 0.1$ m² kg⁻¹

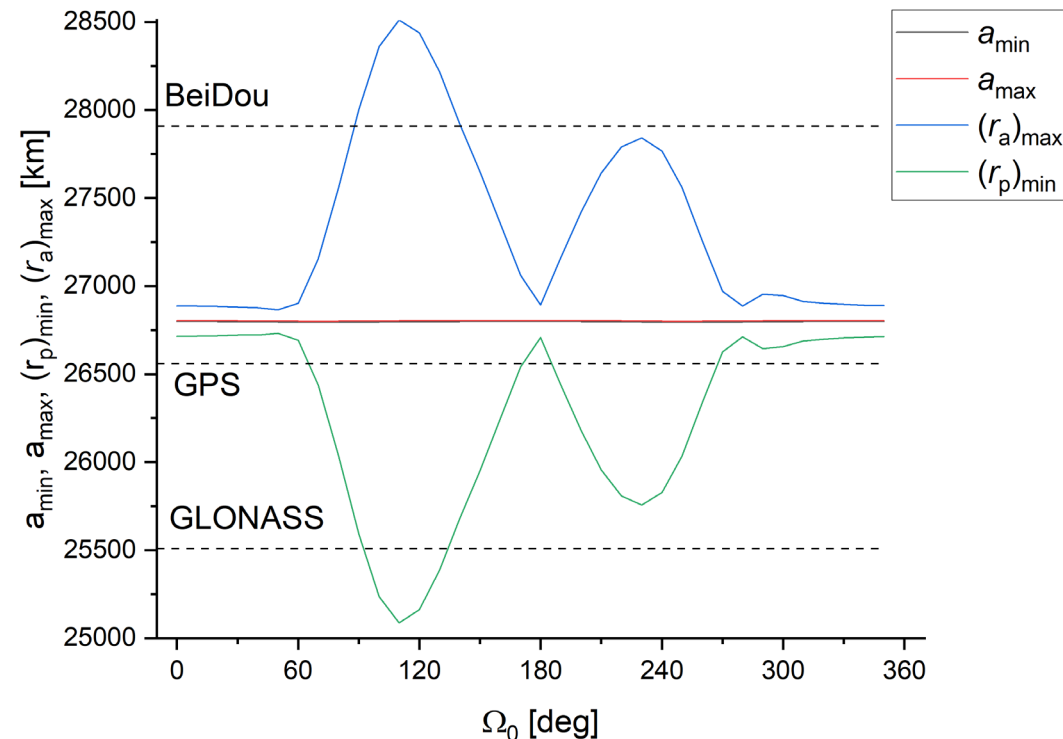


Evolution of $a_{\min}$, $a_{\max}$, $(r_p)_{\min}$, $(r_a)_{\max}$ and resonant angle ψ for $a_0 = 26800$ km (GPS) with $A/m = 0.1$ m² kg⁻¹



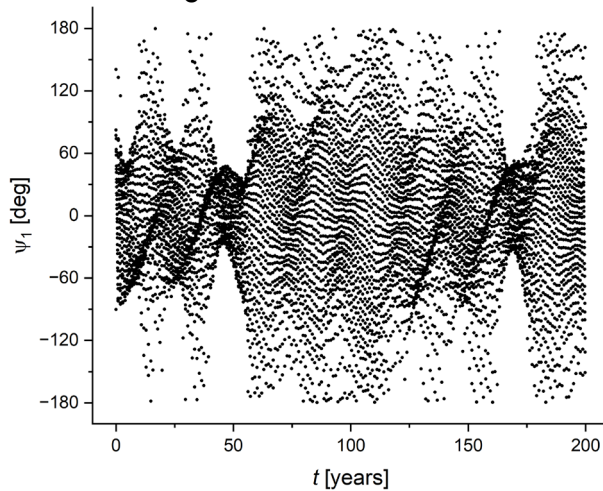
Secular luni-solar resonance

- Resonant angle
 $\psi = 2g + \Omega$

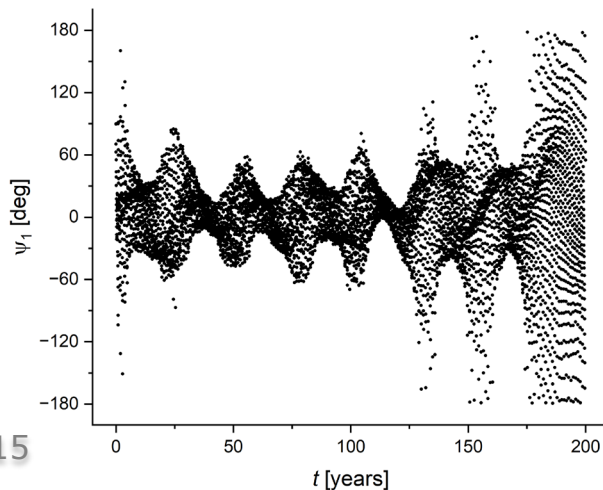


Evolution of $a_{\min}$, $a_{\max}$, $(r_p)_{\min}$, $(r_a)_{\max}$ and resonant angle ψ_1 for $a_0 = 26800$ km (GPS) with $A/m = 0.1$ m² kg⁻¹

$$\Omega_0 = 0^\circ$$



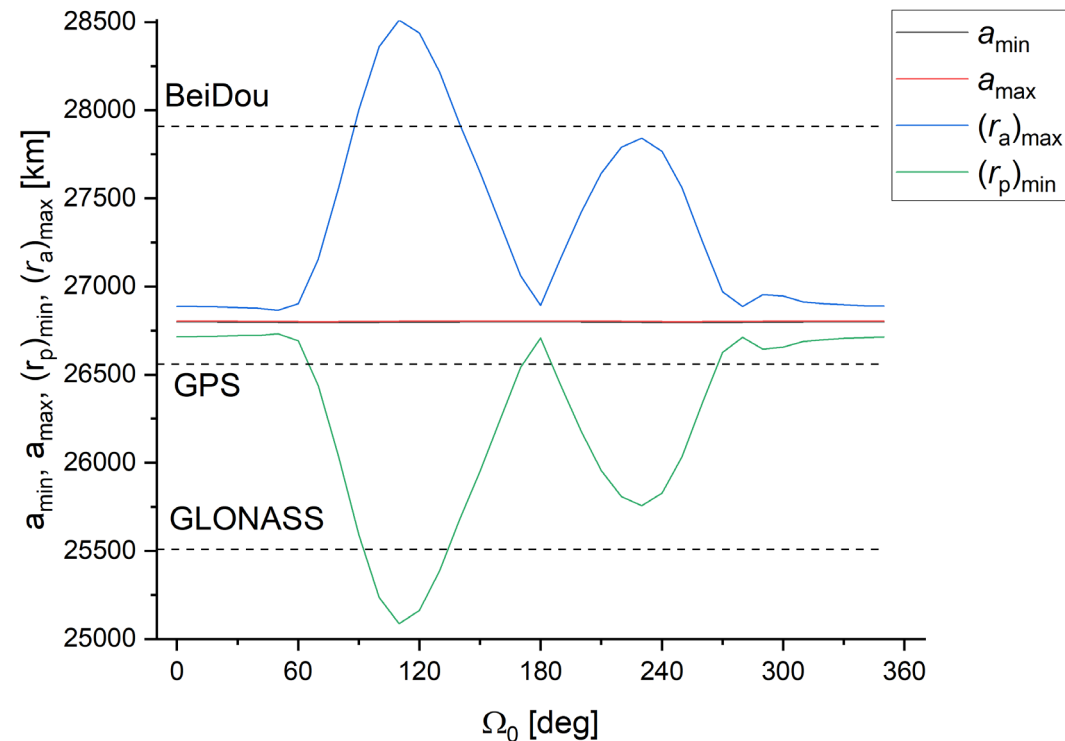
$$\Omega_0 = 180^\circ$$



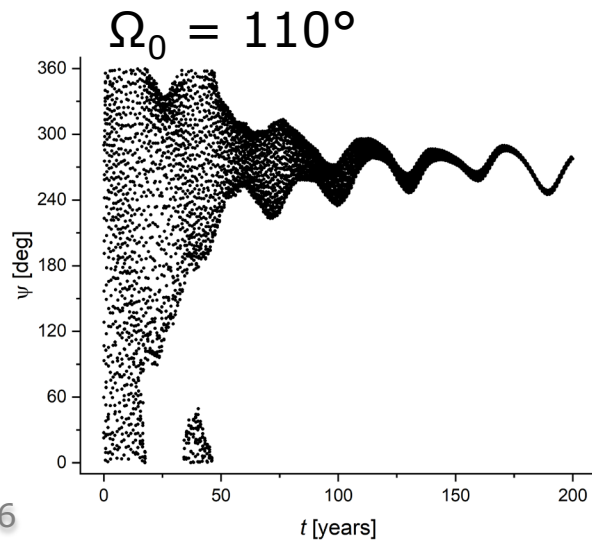
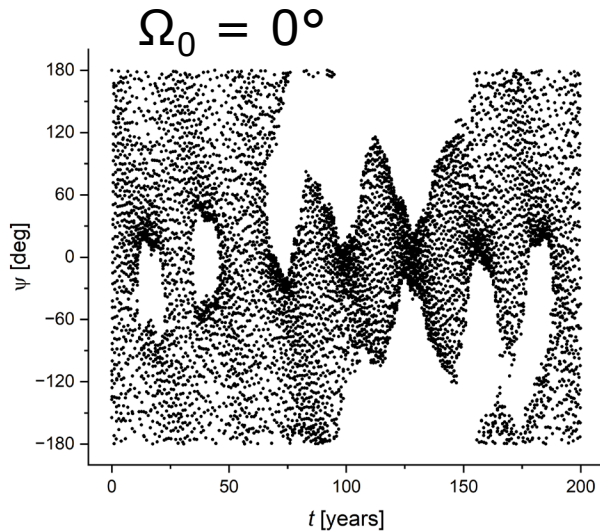
Secondary secular apsidal-nodal resonance

- Resonant angle

$$\psi_1 = \pi - \lambda_S$$

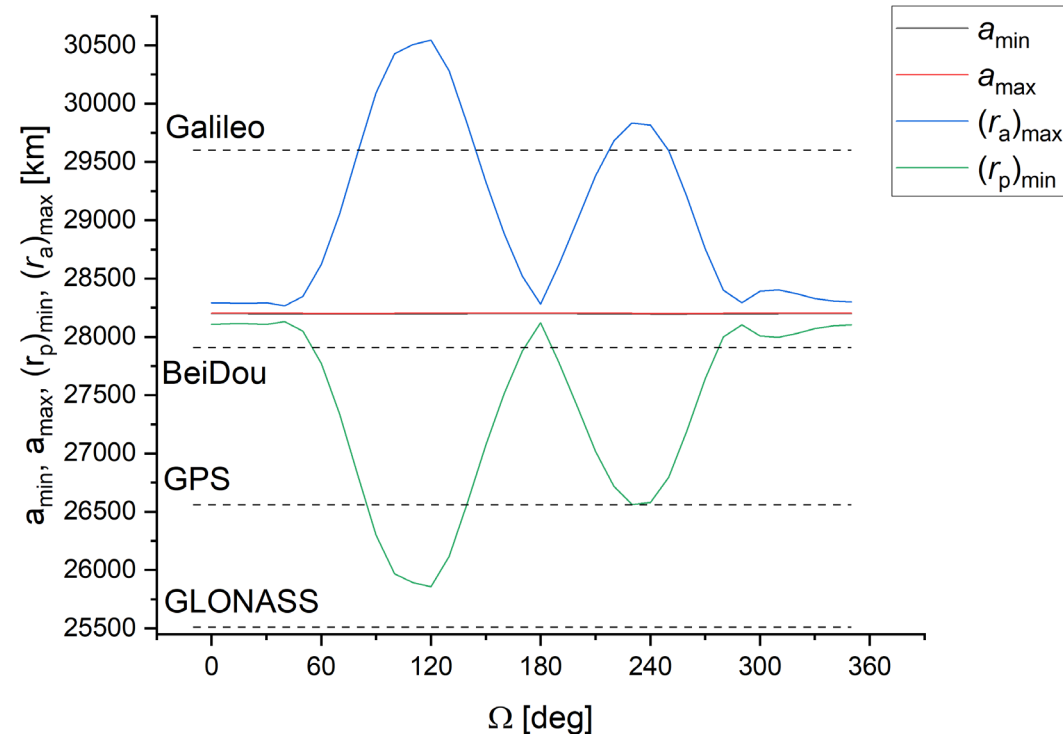


Evolution of $a_{\min}$, $a_{\max}$, $(r_p)_{\min}$, $(r_a)_{\max}$ and resonant angle ψ for $a_0 = 28100$ km (BeiDou) with $A/m = 0.1 \text{ m}^2 \text{ kg}^{-1}$



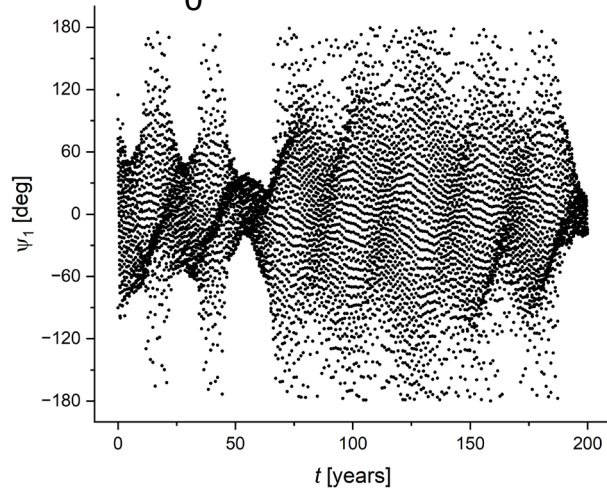
Secular luni-solar resonance

- Resonant angle
 $\psi = 2g + \Omega$

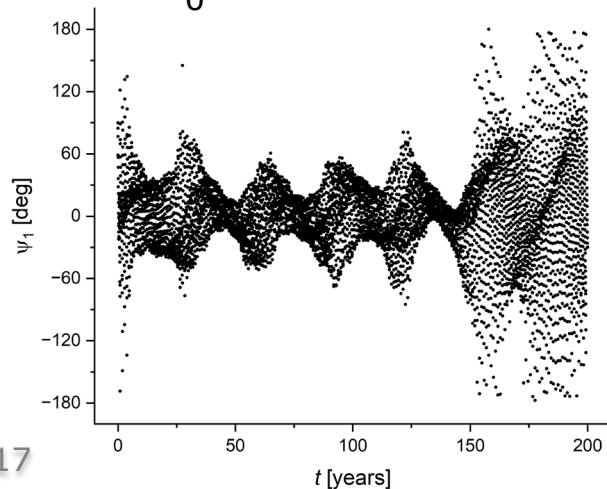


Evolution of $a_{\min}$, $a_{\max}$, $(r_p)_{\min}$, $(r_a)_{\max}$ and resonant angle ψ_1 for $a_0 = 28100$ km (BeiDou) with $A/m = 0.1 \text{ m}^2 \text{ kg}^{-1}$

$$\Omega_0 = 0^\circ$$



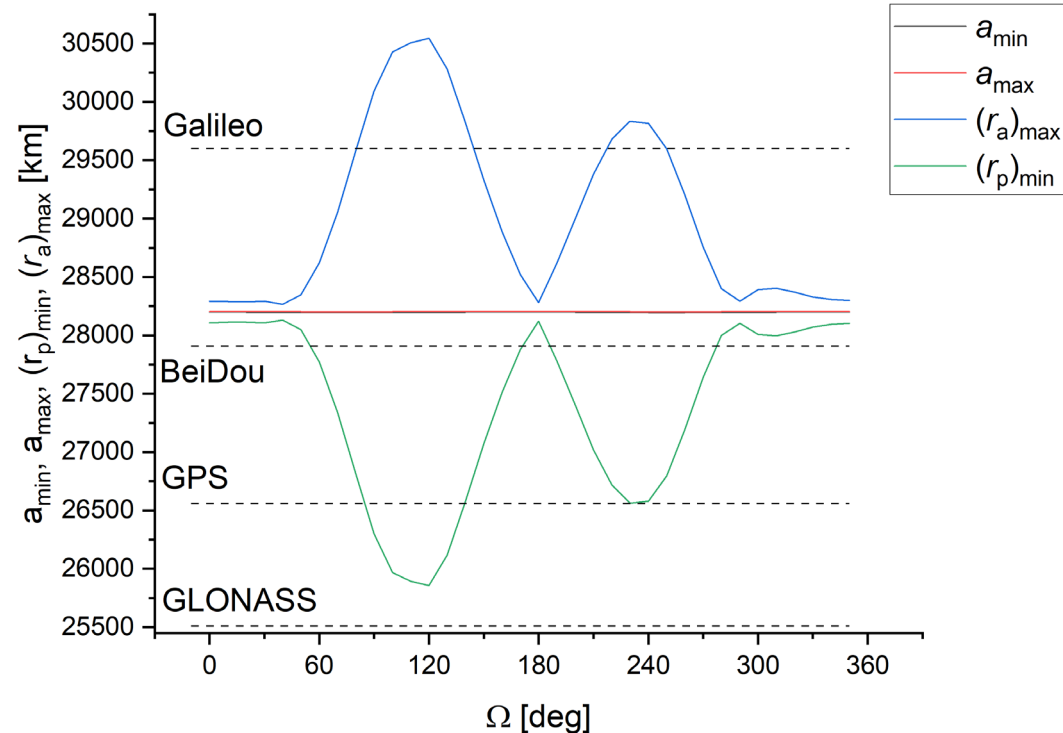
$$\Omega_0 = 180^\circ$$



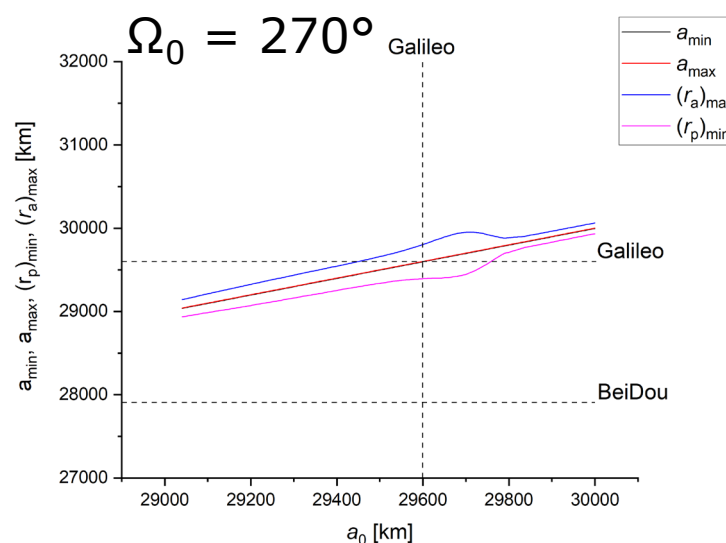
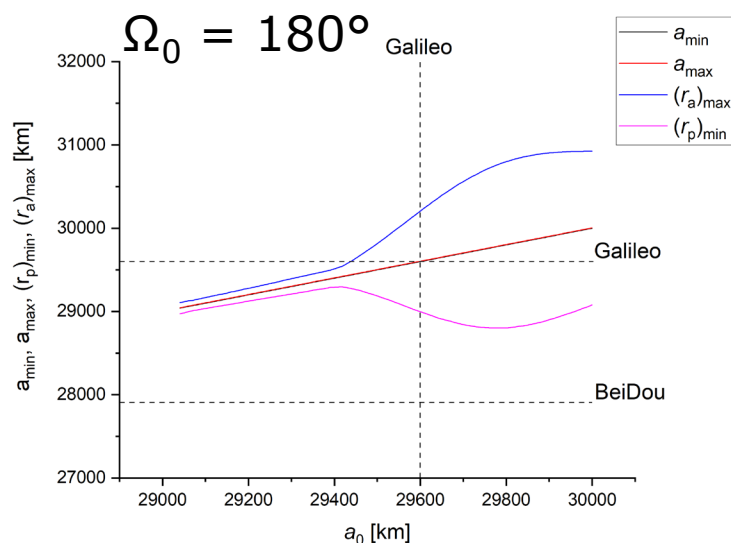
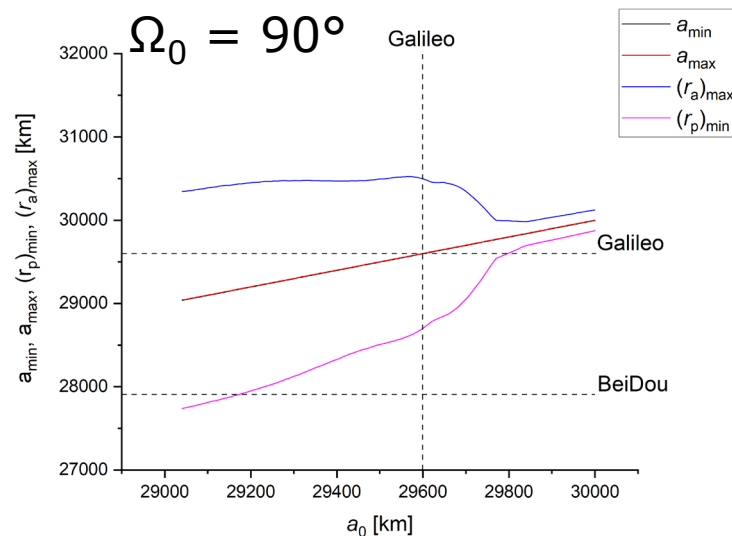
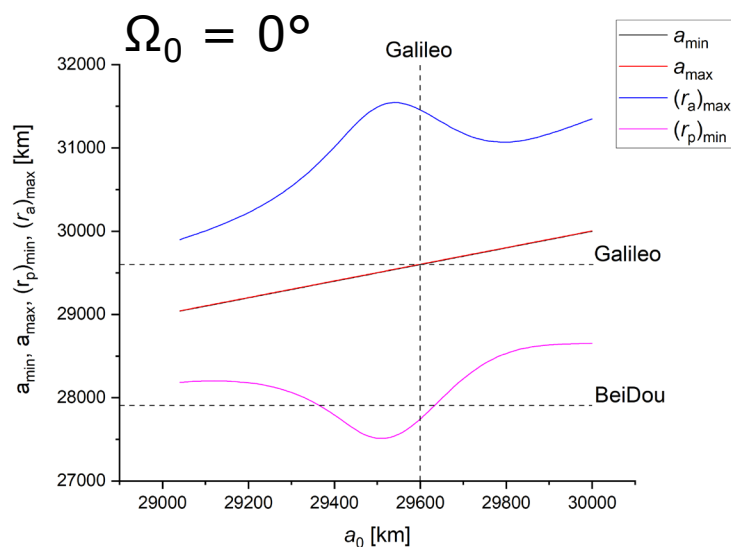
Secondary secular apsidal-nodal resonance

- Resonant angle

$$\psi_1 = \pi - \lambda_S$$



Evolution of the semi-major axis a , distances in perigee r_p and apogee r_a for Galileo region with $A/m = 0.01 \text{ m}^2 \text{ kg}^{-1}$



Initial values of the semi-major axes a_0 [km], at which the object **does not enter** the areas of motion of the satellites of navigational constellations at the argument of periapsis $g_0 = 270^\circ$

GNSS	$\Omega_0 = 0^\circ$	$\Omega_0 = 90^\circ$	$\Omega_0 = 180^\circ$	$\Omega_0 = 270^\circ$
$A/m = 0.01 \text{ m}^2 \text{ kg}^{-1}$				
GLONASS	–	–	25 530–25 950	–
GPS	26 600–27 850	27 030–27 360	27 090–27 470	26 600–27 650
BeiDou	27 960–29 040	28 710–28 760	28 330–29 040	28 270–29 040
Galileo	–	29 800–30 000	–	29 760–30 000
$A/m = 1 \text{ m}^2 \text{ kg}^{-1}$				
GLONASS	–	–	25 850–25 950	–
GPS	–	–	–	–
BeiDou	28 660–28 850	–	–	–
Galileo	–	–	–	29 040–29 110

a_0 [km]	Ω_0 [deg]
Above GLONASS ($i_0 = 64.8^\circ$)	
25 700	110–220
25 800–25 950	100–230
Above GPS ($i_0 = 55.0^\circ$)	
27 100–27 200	0–360
27 300	0–90, 160–360
27 400	0–80, 170–360
27 500	0–70, 200–360
27 600	0–70, 220–360
27 700	0–60, 230–260, 310–360
27 800	0–40, 250, 330–360
Above BeiDou ($i_0 = 55.0^\circ$)	
28 000	0–40, 250, 330–360
28 100	0–50, 240–260, 320–360
28 200	0–50, 220–260, 310–360
28 300	0–60, 210–270, 300–360
28 400	0–70, 160–360
28 500	0–80, 150–360
28 600	0–80, 140–360
28 700–28 900	0–80, 130–360
29 000	0–70, 130–360

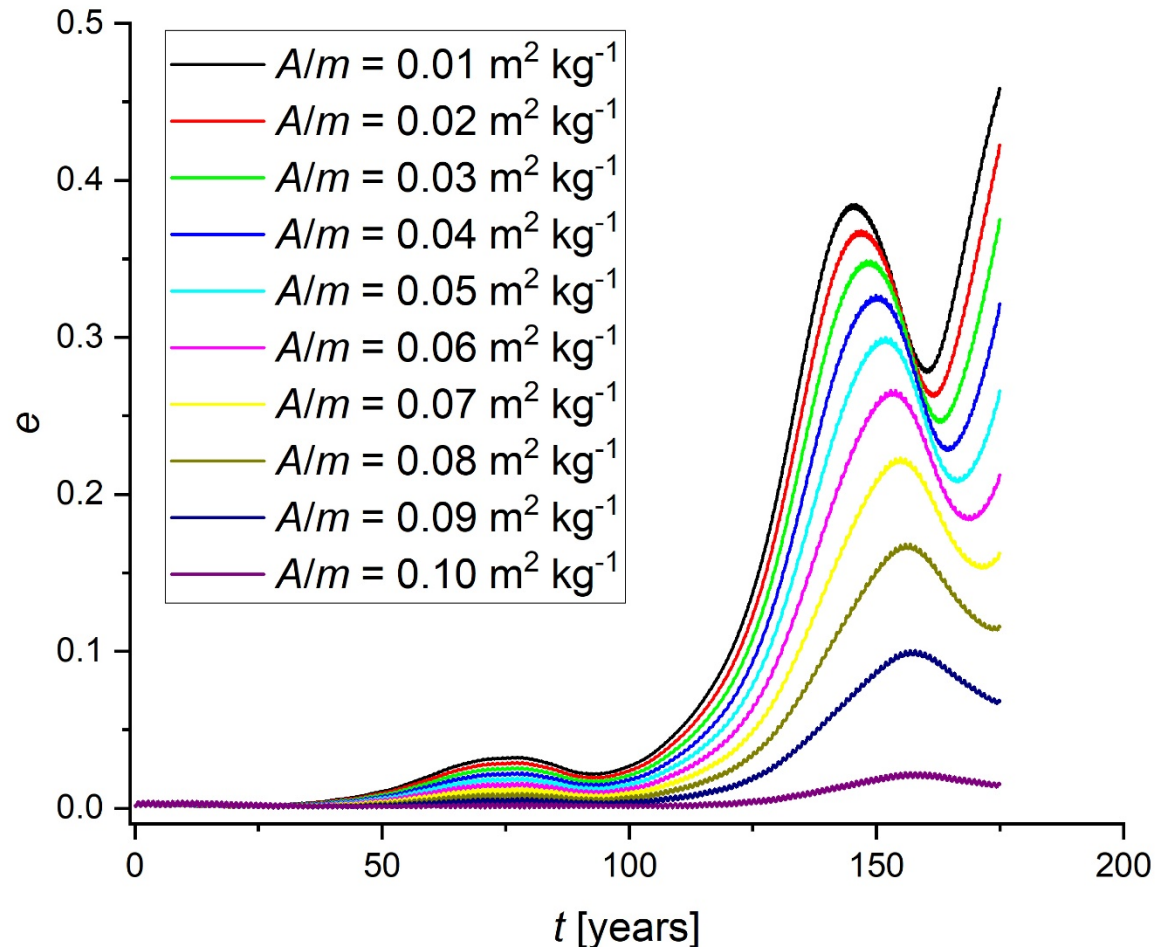
The initial values of the semi-major axes a_0 and the longitudes of the ascending nodes Ω_0 near the GNSS regions that **exclude entry** into the operational areas of navigation satellite constellations ($A/m = 0.01 \text{ m}^2 \text{ kg}^{-1}$)

Evolution of the orbital eccentricity e for the **BDS-IGSO** region

Inversion $\Leftarrow$ Secondary secular apsidal-nodal resonance near stable point

As the area-to-mass ratio A/m **increases**, the maximum value of the eccentricity **e decreases** instead of increasing.

$a_0 = 42\,414$ km,
 $e_0 = 0.0020134$,
 $\Omega_0 = 0^\circ$,
 $g_0 = 270^\circ$

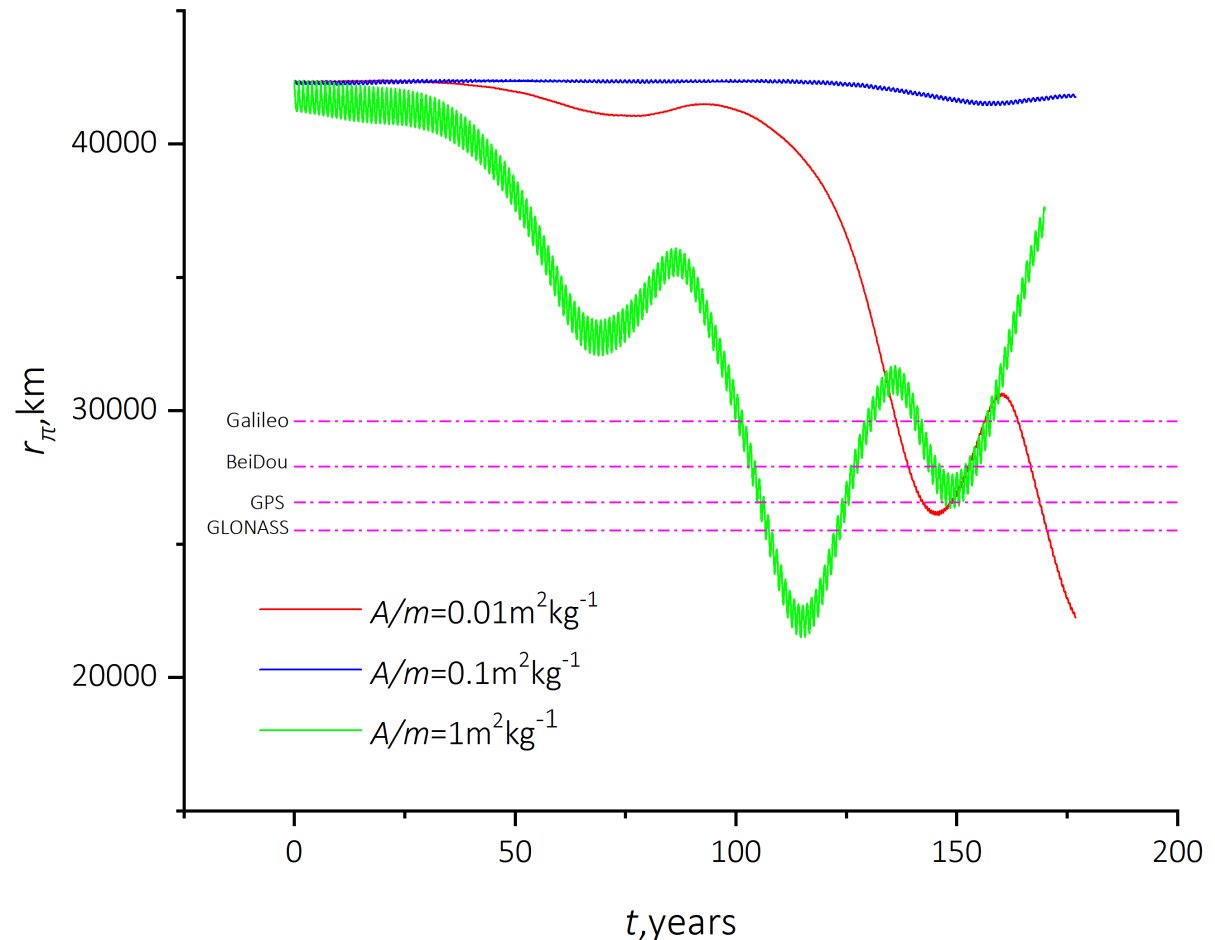


Evolution of the distance in perigee r_p for the **BDS-IGSO** region

Inversion

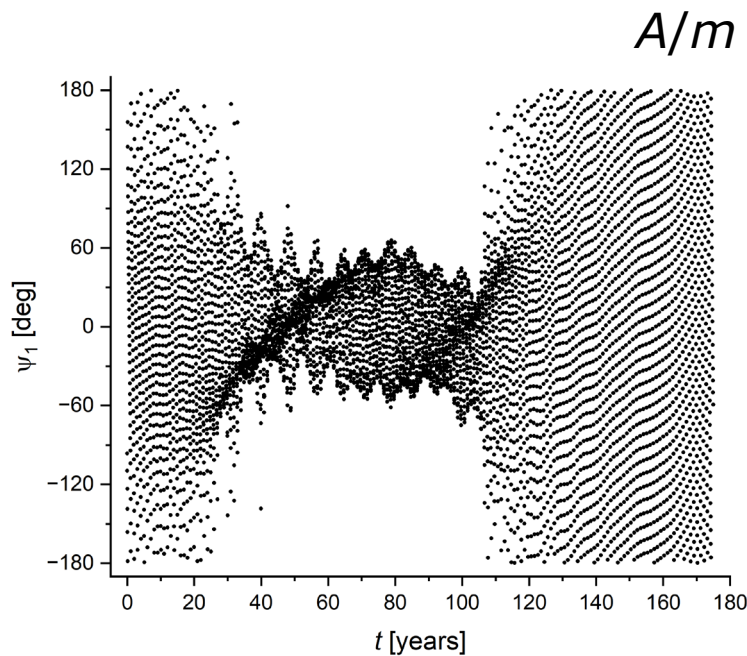
As the area-to-mass ratio A/m **increases**, the maximum value of the eccentricity **e decreases** instead of increasing.

$$\begin{aligned} a_0 &= 42\,414 \text{ km}, \\ e_0 &= 0.0020134, \\ \Omega_0 &= 0^\circ, \\ g_0 &= 270^\circ \end{aligned}$$

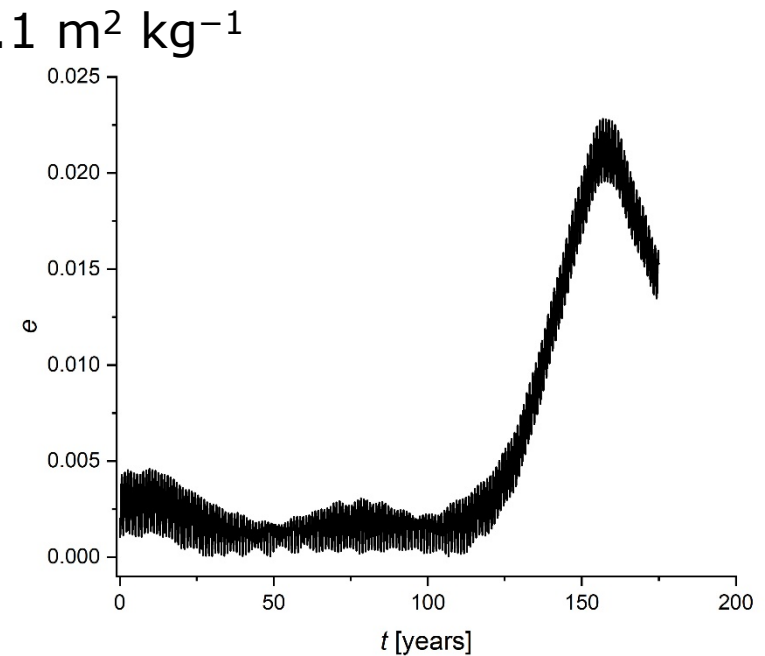


Evolution of the resonant angle ψ_1 and orbital eccentricity e for the **BDS-IGSO** region

Resonant angle ψ_1



Eccentricity e



Secondary secular apsidal-nodal resonance

$$\begin{aligned} a_0 &= 42\,414 \text{ km}, & \Omega_0 &= 0^\circ, \\ e_0 &= 0.0020134 & g_0 &= 270^\circ \end{aligned}$$

- Resonant angle

$$\psi_1 = \pi - \lambda_S$$

Conclusions

- **One of the two resonances**, either **the secular luni-solar resonance** or **the secondary secular apsidal-nodal resonance**, dominates and **determines** the evolution of the orbital eccentricity for a given initial value of the longitude of the ascending node and area-to-mass ratios $A/m \geq 0.01 \text{ m}^2 \text{ kg}^{-1}$.
- **The secondary secular apsidal-nodal resonance** due to solar radiation pressure ensures evolution **in the vicinity of a stationary point**, which **limits the growth of the orbital eccentricity**. **Space debris**, including objects with high ratios A/m , **could potentially accumulate** in the vicinity of this region.
- We propose to transfer **the end-of-life GNSS satellites** into orbits **from 100 to 500 km higher** on the semi-major axis with respect to corresponding operational orbits.
- By suitable **positioning of the orbital plane** with respect to the direction to **the Sun**, it is possible to achieve **a minimal variations in eccentricity** over a long-time interval of **up to 200 years** for area-to-mass ratios $A/m < 1 \text{ m}^2 \text{ kg}^{-1}$.

Thank you for you attention!