

Nonlinear Formation Flying Control of Small Spacecraft in the Vicinity of Asteroid 4179 Toutatis

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Abstract. This study addresses the problem of long-term bounded relative motion maintenance for a spacecraft swarm operating in the vicinity of an asteroid. Two nonlinear control strategies are comparatively investigated: a State-Dependent Riccati Equation (SDRE)-based controller and a Lyapunov-based control approach. The performance of both methods is evaluated in terms of stability, robustness, and control efficiency for spacecraft formation-keeping in the perturbed and uncertain dynamical environment surrounding a small celestial body.

Introduction

Asteroid exploration is challenging due to the small size, irregular geometry, and uncertain gravitational fields of these bodies. Distributed spacecraft formations enable improved spatial coverage, simultaneous observations, and accurate characterization of the asteroid environment, supporting gravity-field reconstruction, autonomous navigation, and future resource utilization[1, 2] .

This study investigates a spacecraft formation near asteroid (4179) Toutatis. Its slow rotation enables coordinated observations and gradual refinement of the gravitational model. A chief spacecraft in a quasi-frozen orbit provides a stable reference trajectory for long-term operations.

To counteract secular drift caused by gravitational perturbations and model uncertainties, two nonlinear control strategies are compared for maintaining bounded relative motion: the State-Dependent Riccati Equation (SDRE) method and a Lyapunov-based control approach [4].

Equations of motion

The relative motion between a chief and a deputy spacecraft is considered. The chief spacecraft defines the reference trajectory by following a prescribed nominal

orbit, which may be either passive or actively controlled. The deputy spacecraft is required to maintain a bounded relative trajectory with respect to the chief.

Due to the highly perturbed and uncertain gravitational environment of the asteroid, secular drift occurs in the relative motion, requiring active formation-keeping control. Both control strategies are designed using an approximate relative dynamics model formulated in terms of relative orbital elements [5]. The averaged equations describing the motion near the chief spacecraft's quasi-frozen orbit are derived from the Lagrange planetary equations and given as follows:

$$\begin{aligned}
\dot{\bar{a}} &= 0, \\
\dot{\bar{e}} &= 0, \\
\dot{\bar{i}} &= \frac{3\bar{n}}{(\bar{p}/R_0)^2} C_{22} \sin \bar{i} \sin 2\bar{\Omega}_R, \\
\dot{\bar{\Omega}}_R &= \frac{3\bar{n} \cos \bar{i}}{2(\bar{p}/R_0)^2} (C_{20} + 2C_{22} \cos 2\bar{\Omega}_R) - n_T, \\
\dot{\bar{\omega}} &= -\frac{3\bar{n}}{8(\bar{p}/R_0)^2} [C_{20} (3 + 5 \cos 2\bar{i}) + 2C_{22} \cos 2\bar{\Omega}_R (-1 + 5 \cos 2\bar{i})], \\
\dot{\bar{M}} &= \bar{n} - \frac{3\bar{n} \sqrt{1 - \bar{e}^2}}{8(\bar{p}/R_0)^2} [C_{20} (1 + 3 \cos 2\bar{i}) + 6C_{22} \cos 2\bar{\Omega}_R (\cos 2\bar{i} - 1)].
\end{aligned} \tag{1}$$

The notation of these elements (where the overbar denotes the averaged value of the corresponding quantity) is as follows: $\bar{a}$ is semi-major axis, $\bar{e}$ stands for eccentricity, $\bar{i}$ is orbital inclination, $\bar{\Omega}$ the longitude of the ascending node, $\bar{\omega}$ denotes argument of periapsis, $\bar{M}$ is mean anomaly, C_{20} is zonal harmonic coefficient, and C_{22} is sectoral harmonic coefficient, and defining orbital element vector $\boldsymbol{\alpha}$ is:

$$\boldsymbol{\alpha} = [\bar{a}, \bar{e}, \bar{i}, \bar{\Omega}, \bar{\omega}, \bar{M}]^T.$$

Control of motion

the dynamical system with control is given:

$$\dot{\boldsymbol{\alpha}}_D = \mathbf{f}(\boldsymbol{\alpha}_D) + \mathbf{B}(\boldsymbol{\alpha}_D) \mathbf{u}_D + \boldsymbol{\delta} \tag{2}$$

where the system $\mathbf{f}(\boldsymbol{\alpha}_D)$ is defined by the following equation, $\boldsymbol{\delta}$ represents the dominant disturbances, and $\mathbf{B}(\boldsymbol{\alpha}_D)$ is the control input matrix:

$$\mathbf{B}(\boldsymbol{\alpha}_D) = \begin{bmatrix} \frac{2e \sin \nu}{n\sqrt{1-e^2}} & \frac{2(1+e \cos \nu)}{n\sqrt{1-e^2}} & 0 \\ \frac{\sqrt{1-e^2} \sin \nu}{na} & \frac{\sqrt{1-e^2}(e+2 \cos \nu + e \cos^2 \nu)}{na(1+e \cos \nu)} & 0 \\ 0 & 0 & \frac{\sqrt{1-e^2} \cos(\omega + \nu)}{na(1+e \cos \nu)} \\ 0 & 0 & \frac{\sqrt{1-e^2} \sin(\omega + \nu)}{na \sin i(1+e \cos \nu)} \end{bmatrix} \quad (3)$$

where ν – is the true anomaly. The SDRE approach [3] synthesizes a feedback controller by transforming the nonlinear system into a state-dependent linear form and solving a Riccati equation online at each control step. This provides a near-optimal control solution with respect to the selected performance criterion but requires significant computational resources.

In contrast, the Lyapunov-based approach derives the control law analytically by selecting a positive-definite Lyapunov function and ensuring a negative-definite time derivative, which guarantees closed-loop stability [4]. This method offers a computationally efficient and robust solution, although it is generally not optimal with respect to a predefined performance index.

The proposed nonlinear control architecture combines a nominal stabilization controller with an adaptive compensation mechanism. The nominal controller, based on either the SDRE or Lyapunov approach, ensures bounded relative motion of the deputy spacecraft, while the adaptive component compensates for parametric uncertainties and unmodeled disturbances, enhancing robustness against dynamical model imperfections.

Conclusion

This study addresses the problem of long-term bounded relative motion maintenance for a spacecraft swarm operating in the vicinity of an asteroid. Numerical simulations compare the performance of the two control strategies in terms of stability, robustness, and control effort. The results provide practical insights into the suitability of each approach for maintaining distributed spacecraft formations in the highly uncertain dynamical environment surrounding small celestial bodies. Furthermore, the proposed methodology can be readily extended to centralized control architectures for large swarms of small spacecraft operating around asteroids.

References

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