

Using orbital dynamics stability criteria in the search for exomoons

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Abstract. Within the framework of the exoplanet satellite search, we performed a comparative analysis of the applicability of various criteria for determining the size of a stable satellite orbit.

Introduction

One of the pressing tasks of modern astronomy is the search for exoplanet satellites (exomoons) [3, 4]. The long-term orbital dynamics of a satellite that actually exists in a planetary system must be stable. Numerical modeling, namely the study of motion stability allows us to assess the possibility of detecting exomoons from the analysis of observations and to obtain/refined information on the orbital and physical parameters of identified exomoon candidates. By calculating the Lyapunov characteristic exponents (LCEs; see [7] for details) in the hierarchical three-body problem (“star – planet – planetary satellite”), we analyzed the applicability of several criteria [1, 2, 8, 10] for estimating the maximum semi-major axis of a satellite’s orbit a_{crit} , corresponding to its long-term stable orbital dynamics.

Stability diagrams

To evaluate the applicability of empirical stability criteria to the study of exoplanet satellite dynamics, we calculated the Lyapunov times (the timescale of predictable dynamics) of the exomoon’s orbital dynamics over sets of initial conditions typical for this problem: (a_s, i_s) , (a_s, e_p) , and (a_s, e_s) . The orbital elements of the satellite subsystem are denoted by the subscript “s”, and those of the planetary system by the subscript “p”. Based on these sets of initial conditions, the boundaries of the stable orbital dynamics regions were determined, and the corresponding stability diagrams were constructed. Figure 1 shows examples of the constructed stability diagrams for the exoplanet satellite system. The semi-major axis of the

satellite's orbit a_s is given in units of the planet's Hill sphere radius R_H . The diagrams indicate the Lyapunov time T_L of the planet's satellite subsystem. The Lyapunov time was estimated as [9] $T_L = 1/L$, where L is the maximum LCE. The methodology for constructing the stability diagrams is described in [5, 6].

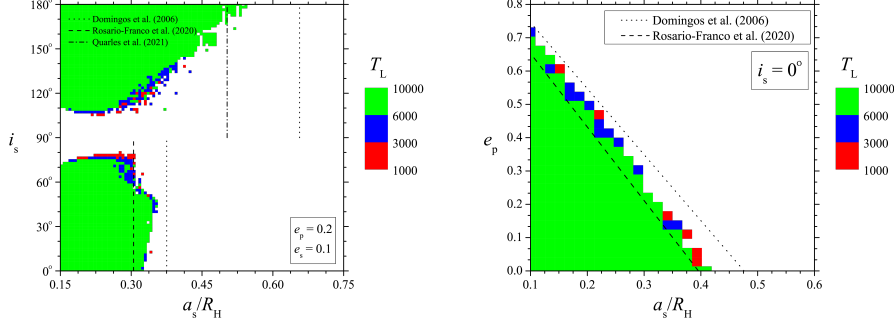


FIGURE 1. Lyapunov times (in years) of the exoplanet satellite subsystem calculated over a set of initial conditions: (a_s, i_s) for $(e_p = 0.2, e_s = 0.1)$ and (a_s, e_p) for $e_s = 0, i_s = 0$. White color corresponds to unstable exomoon orbits. The dashed, dotted, and dash-dotted lines indicate the empirical criteria considered [2, 8, 10] for the maximum value of a_s corresponding to the stable dynamics of the planetary satellite.

Conclusion

Analysis of the constructed stability diagrams showed that the empirical criterion proposed by Domingos et al. [2], frequently used to determine potentially possible exomoon orbits, significantly overestimates the theoretically possible value of a_s . In the coplanar problem for a prograde exomoon orbit (where the directions of the planet's and satellite's orbital motions coincide), the criterion proposed by Rosario-Franco et al. [8] performs better when estimating a_{crit} , while for a retrograde exomoon orbit, the criterion from [10] is more accurate. If the angle between the orbital planes of the planet and the satellite can be substantial ($i_s \geq 30^\circ$), a reliable estimate of a_{crit} can only be obtained through numerical simulation of the long-term orbital dynamics of the exomoon and by employing rigorous motion stability analysis methods (such as calculating LCEs, the MEGNO parameter, etc.).

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