

Total Effect of the Earth's Gravitational Field on the Tide-Generating Potential

Sergey M. Kudryavtsev

Extended abstract

The Earth tide-generating potential (TGP) is one of the most important potentials of celestial mechanics. It is used, e.g., in theories of Earth nutation, developments of solid and ocean tides, analysis of geodetic and gravimetric observations, etc.

In the current presentation exact formulae are given for calculating a new part of the Earth TGP caused by a joint action of external gravitating bodies (the Moon, the Sun, major planets, ...) and an *arbitrary* spherical harmonic of the geopotential expansion. This extends the results of the previous studies [1, 2, 3], where only the effect of the second-degree geopotential coefficients was considered. The formulae explicitly separate some time variables (named the “TGP coefficients”), which are functions of the geopotential harmonics, masses and instantaneous positions of the external bodies, from the geographical coordinates of an observation point on (or above) the Earth's surface, where the TGP is evaluated.

By using these formulae we calculate the TGP coefficients over a time interval of 30,000+ years (-13,200–17,191) with a small sampling step of one day and then expand them in accurate harmonic series. The modern values for the masses and instantaneous positions of the Moon/the Sun/major planets provided by the recent long-term numerical ephemeris DE441 [4] are used as the source. An author's modification of the spectral analysis method [5, 6] is applied as the expansion tool. It allows one to represent an arbitrary tabulated function of planetary/lunar coordinates by a generalized harmonic series, where both amplitudes and arguments of the series' terms are high-degree polynomials of time.

As a result, new terms in the Earth TGP development are found. They are due to the geopotential coefficients of the third degree. The minimum amplitude of terms in the obtained harmonic series was chosen equal to $1 \times 10^{-9} \text{ m}^2/\text{s}^2$. It is by an order of magnitude less than the similar threshold employed by the latest TGP catalogues HW95 [7] and KSM03 [5]. The use of the thus decreased threshold and the modern planetary/lunar ephemeris allowed us to also improve the already

known terms in the Earth's TGP harmonic expansion caused by the geopotential coefficients of the second degree and increase their total number by a factor of two.

The new and updated terms are available on the web-server of SAI MSU. They are presented in two formats: KSM25 (http://lnfm1.sai.msu.ru/neb/ksm/tgp/k25gp_format-ksm25.dat, see the format description on http://lnfm1.sai.msu.ru/neb/ksm/tgp/readme_format-ksm25.pdf) and a generalized HW95 (http://lnfm1.sai.msu.ru/neb/ksm/tgp/k25gp_format-hw95gen.dat). The former format allows one to represent the TGP coefficients over long intervals of time (thousands and tens of thousands of years), and the latter is currently used in some popular software packages for processing tidal measurements, e.g., ETERNA [8].

A new type of the “permanent tides” is revealed (the permanent tides on the Earth's surface are caused by the constant terms in the harmonic developments of some TGP coefficients). It is originated from the tesseral geopotential coefficients of order 1 (mostly, c_{31} , s_{31}), and the corresponding permanent tides depend on not only the geographical latitude of the observation site (like any other permanent tide found before), but also on the site longitude.

References

- [1] H. Wilhelm, *Earth's flattening effect on the tidal forcing field*, J. Geophys., 52, 131–135 (1983), <https://journal.geophysicsjournal.com/JofG/article/view/193>
- [2] F.A. Dahlen, *Effect of the Earth's ellipticity on the lunar tidal potential*, Geophys. J. Int., 113, 250–251 (1993), <https://doi.org/10.1111/j.1365-246X.1993.tb02543.x>
- [3] S.M. Kudryavtsev, *Effect of the Earth's triaxiality on the tide-generating potential*, J. Geodesy, 99:9 (2025), <http://doi.org/10.1007/s00190-024-01926-2>
- [4] R.S. Park, W.M. Folkner, J.G. Williams, and D.H. Boggs, *The JPL planetary and lunar ephemerides DE440 and DE441*, Astron. J., 161:105 (2021), <https://doi.org/10.3847/1538-3881/abd414>
- [5] S.M. Kudryavtsev, *Improved harmonic development of the Earth tide-generating potential*, J. Geodesy, 77, 829–838 (2004), <https://doi.org/10.1007/s00190-003-0361-2>
- [6] S.M. Kudryavtsev, *Long-term harmonic development of lunar ephemeris*, Astron. Astroph., 471, 1069–1075 (2007), <https://doi.org/10.1051/0004-6361:20077568>
- [7] T. Hartmann and H.-G. Wenzel, *The HW95 tidal potential catalogue*, Geophys. Res. Lett., 22, 3553–3556 (1995), <https://doi.org/10.1029/95GL03324>
- [8] H.-G. Wenzel, *The nanogal software: Earth tide data preprocessing package ETERNA 3.30*, Bull. Inf. Mar. Terrest., 124, 9425–9439 (1996), <http://igets.u-strasbg.fr/BIM/bim124.pdf>

Sergey M. Kudryavtsev
 Department of Celestial Mechanics
 Sternberg Astronomical Institute, Lomonosov Moscow State University
 Moscow, Russia
 e-mail: ksm@sai.msu.ru