

The empiric integral of motion in 3:2 resonance

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Introduction

Here we report about the empiric quasi-integral of motion, bounded proper orbital elements (proper frequency g , eccentricity and semiajor axis) for a special case 3:2 resonance with Jupiter.

Data were taken from the AstDys website: <https://newton.spacedys.com/astdys/> The proper elements of **1668 numbered asteroids** of the Hilda group were used.

The dependences between each pairs of orbital elements of known members of Hilda group are approximated. On base these approximations, the expression, contained a , e , and g is obtained.

Known integrals

Broz and Vokrouhlicky note that motion obeys an integral of motion N given by the formula:

$$N = \sqrt{a} \left(\frac{3}{2} - \sqrt{1 - e^2} \cos i \right)$$

(Ferraz-Mello 1988) **[4]** bounded Delaunau variables and two types of resonant arguments.

4. Ferraz-Mello S. The high-eccentricity libration of the Hildas. AJ, Vol 96, No 1, (1988), p.400-408

Consider the restricted three body problem Sun-Jupiter-asteroid. The mean motion resonance takes place under the following condition on the resonance argument (for the planar case):

$$\phi = j_1 \lambda_p + j_2 \lambda + j_3 \varpi_p + j_4 \varpi \quad (1)$$

with the D'Alembert rule:

$$j_1 + j_2 + j_3 + j_4 = 0$$

Here $\lambda_p, \lambda, \varpi_p, \varpi$ are the mean longitudes, perihelion longitudes of the planet and asteroid respectively, $j_1 \dots j_4$ are integer. For the 3:2 resonance ($j_1 = 3j_2 = -2j_4 = -1$):

$$\dot{\phi} = 3\dot{\lambda}_p - 2\dot{\lambda} - \dot{\varpi} = 0 \quad (2)$$

Approximation: $g = 109296\delta_a + 52.253$

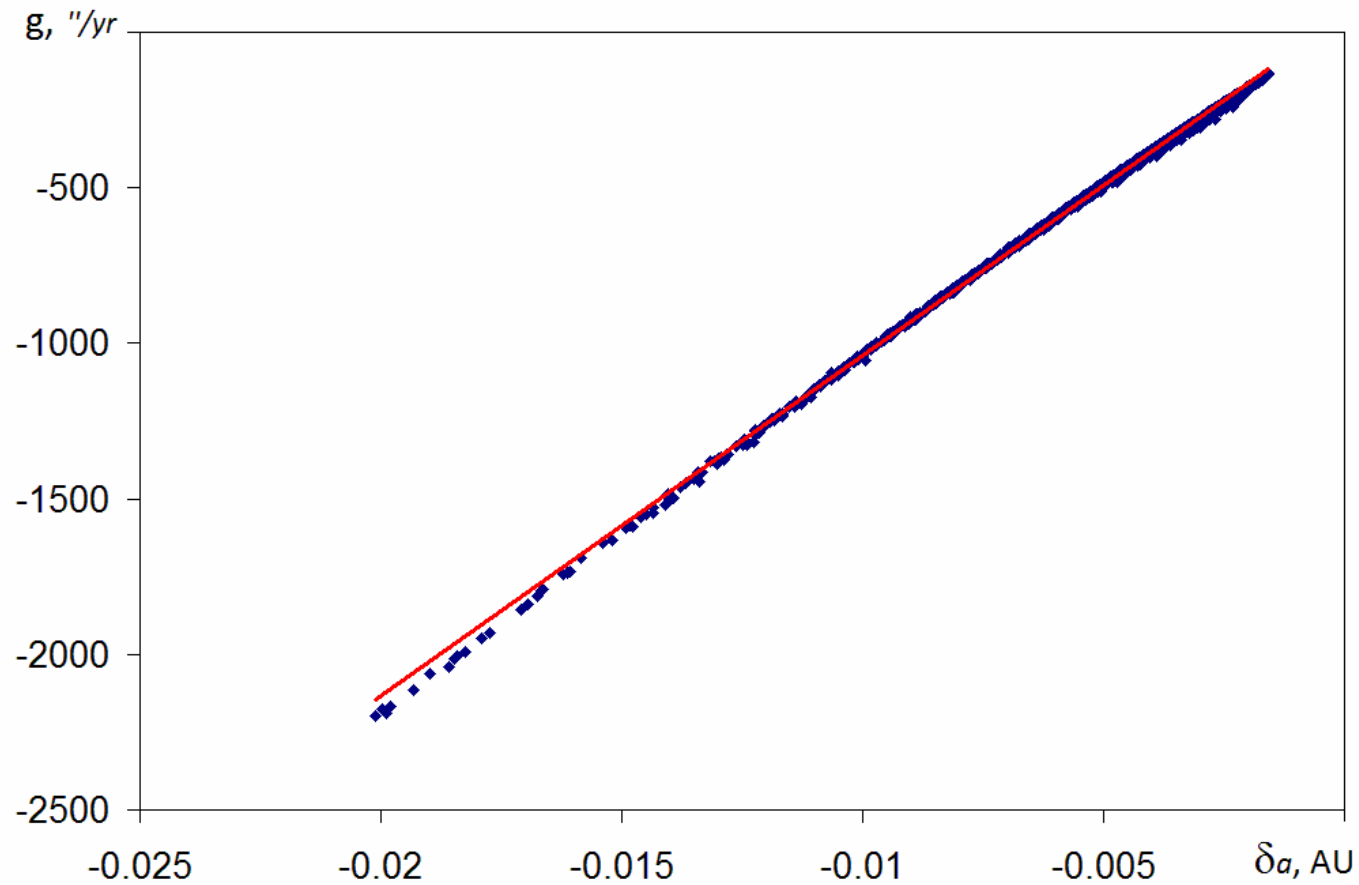


Fig. 1. The proper frequency on the proper semimajor axis dependence

Approximation: $e = 1/(2.8 - 590 * \delta_a)$

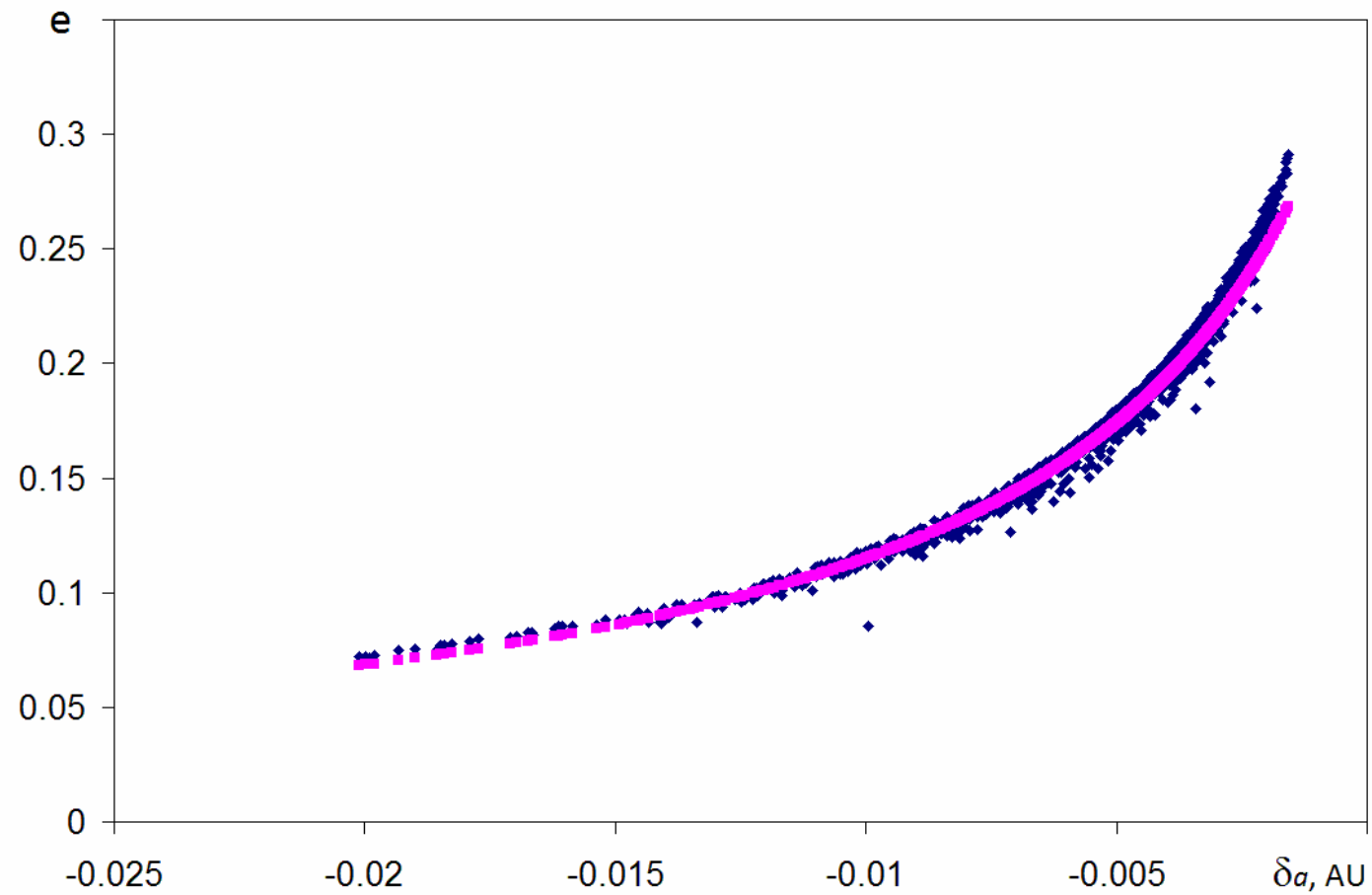


Fig. 2. The proper eccentricity on the proper semimajor axis dependence

Approximation:

$$g = -165.81/e + 450$$

$$g = -49/e^{3/2} + 190$$

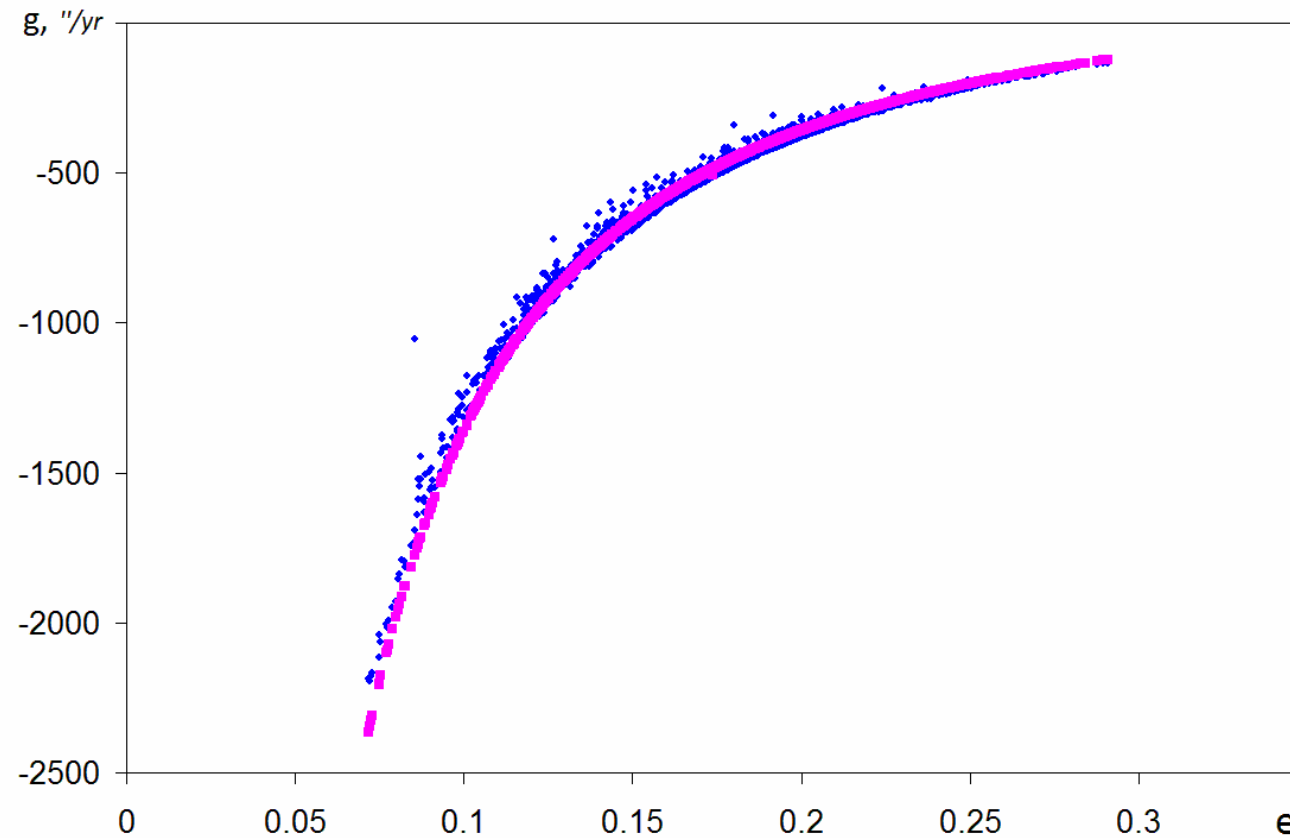
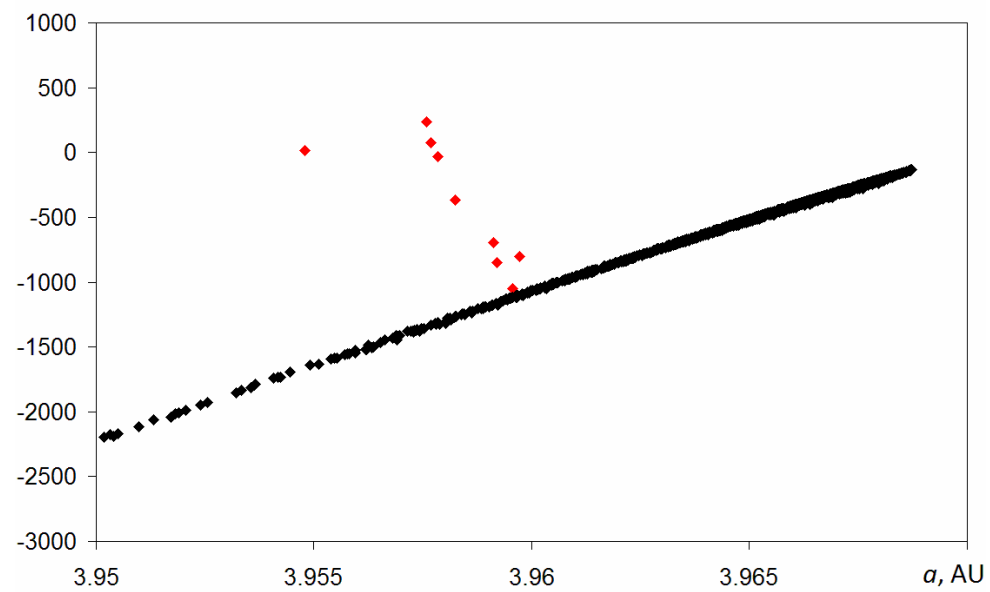
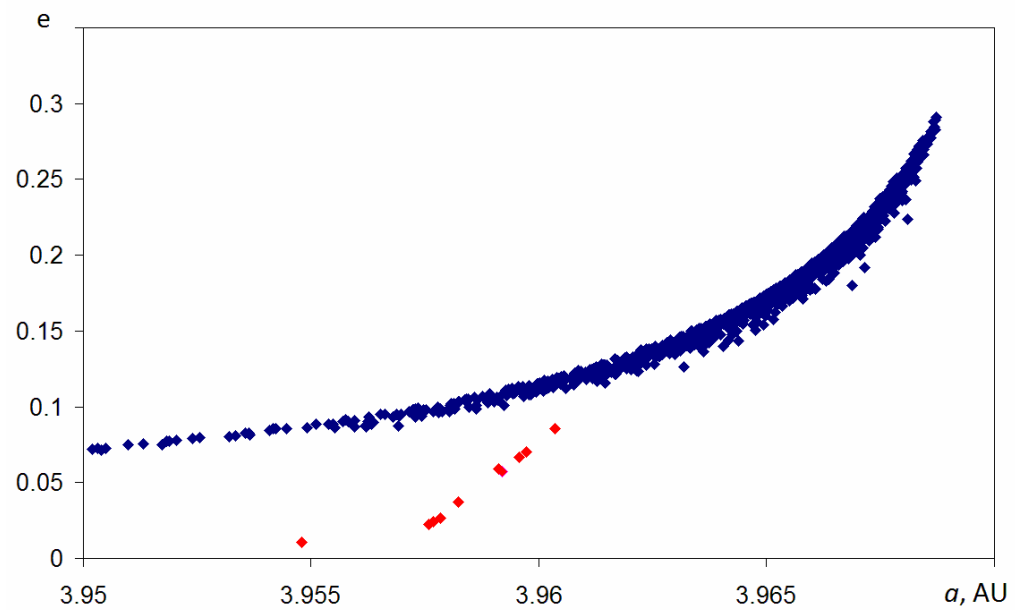


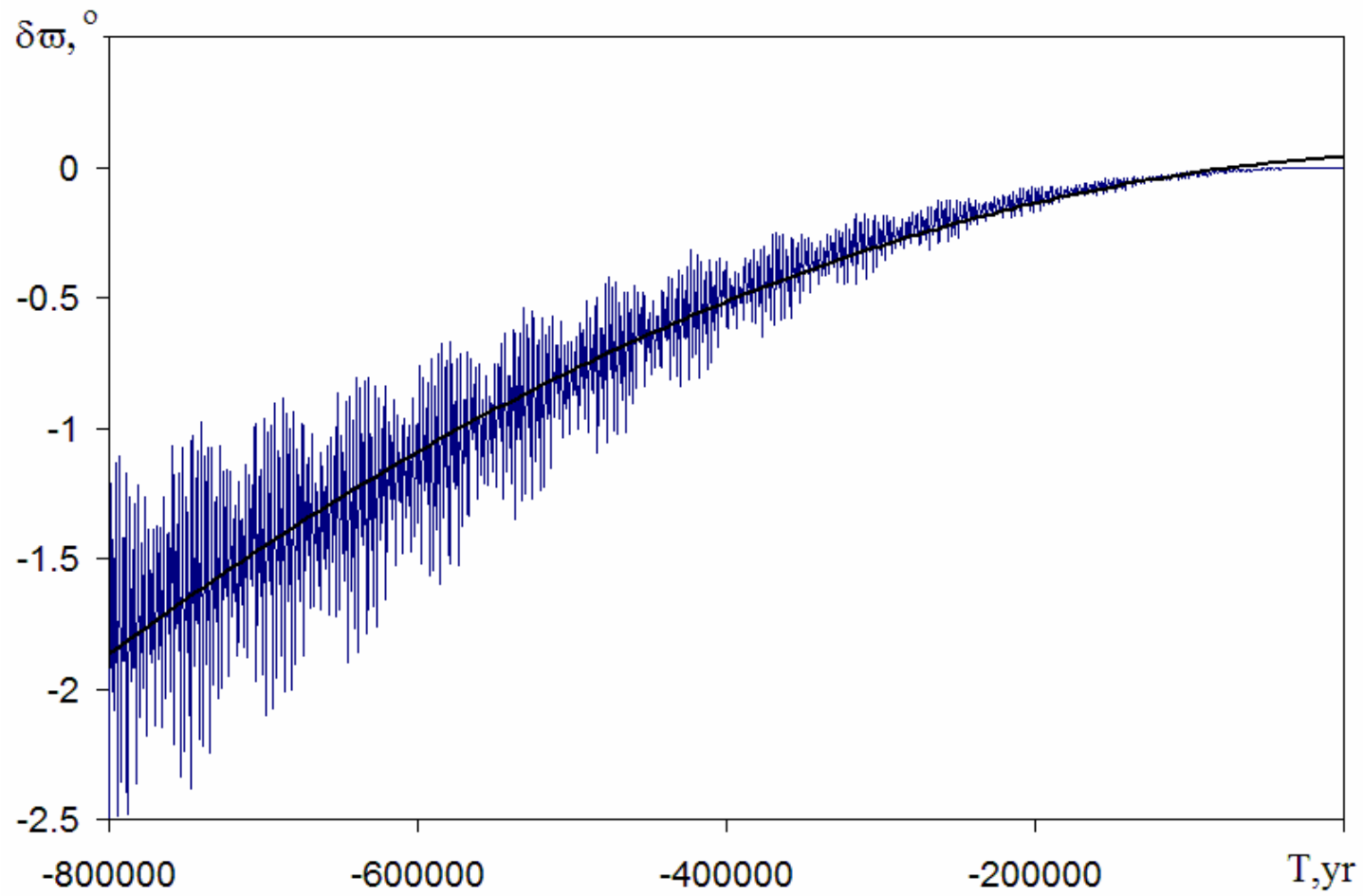
Fig. 3. The proper frequency on the proper eccentricity dependence

Exceptional case of asteroid not included in dependences

NN	a	e	Sin i	g	s
55347	3.954786	0.010412	0.156929	15.17962	-124.81
111995	3.957689	0.024253	0.122048	77.07686	-123.49
200040	3.959565	0.066497	0.201751	-1047.38	-96.346
242996	3.959202	0.05737	0.184827	-850.205	-100.4
328445	3.957841	0.026781	0.155776	-34.9673	-111.75
329241	3.959112	0.059117	0.210272	-694.795	-103.51
374435	3.959728	0.070143	0.212741	-802.658	-97.935
378566	3.957576	0.022178	0.093871	236.429	-125.16
396155	3.958241	0.036946	0.100489	-369.199	-149.23



Yarkovsky effect on g



Numeric integration of fictive asteroids

To study the dynamic evolution of the test particle, the equations of motion of the systems were numerically integrated with orbits over 800 kyr using the N-body integrator Mercury (Chambers 1999) and the Everhart integration method (Everhart 1985).

The initial values of eccentricity (at the epoch 06.07.1998) are given in Table below, when the other orbital elements are the same as for 153 Hilda.

e	a	e	a
0.043	3.96525	0.043	3.97725
0.073	3.96525	0.073	3.97725
0.153,	3.96525	0.153	3.97725
0.173	3.96525	0.173	3.97725
0.203	3.96525	0.203	3.97725

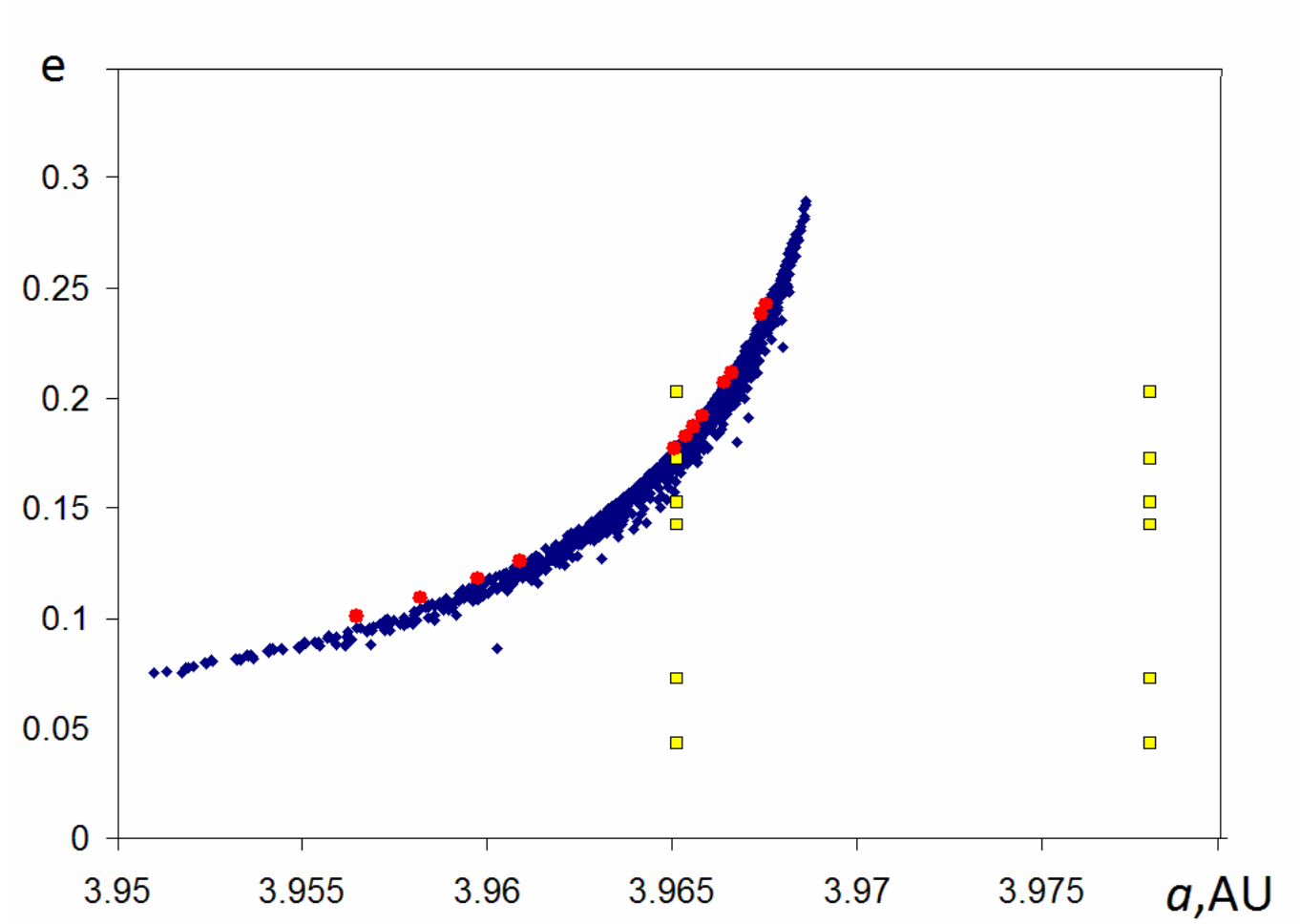


Fig. 4.

Symbolic form

$$g = 109296\delta_a + 52.253 \Rightarrow g = n_J \delta_a + 52.253$$

$$g = -165.81 / e + 450 \Rightarrow g = -n_{res} / e + 450$$

$$e = 1 / (2.8 - 590 * \delta_a) \Rightarrow e = e_{\min} / (e_{\max} - k\delta_a)$$

$$g = -49 / e^{3/2} + 190 \Rightarrow ?$$

The empiric integrals

$$I_1 = 2g - 109296\delta_a + 165.81/e \approx 502.2$$

$$I_2 = 2g - 109296\delta_a + 49/e^{3/2} \approx 242.2$$

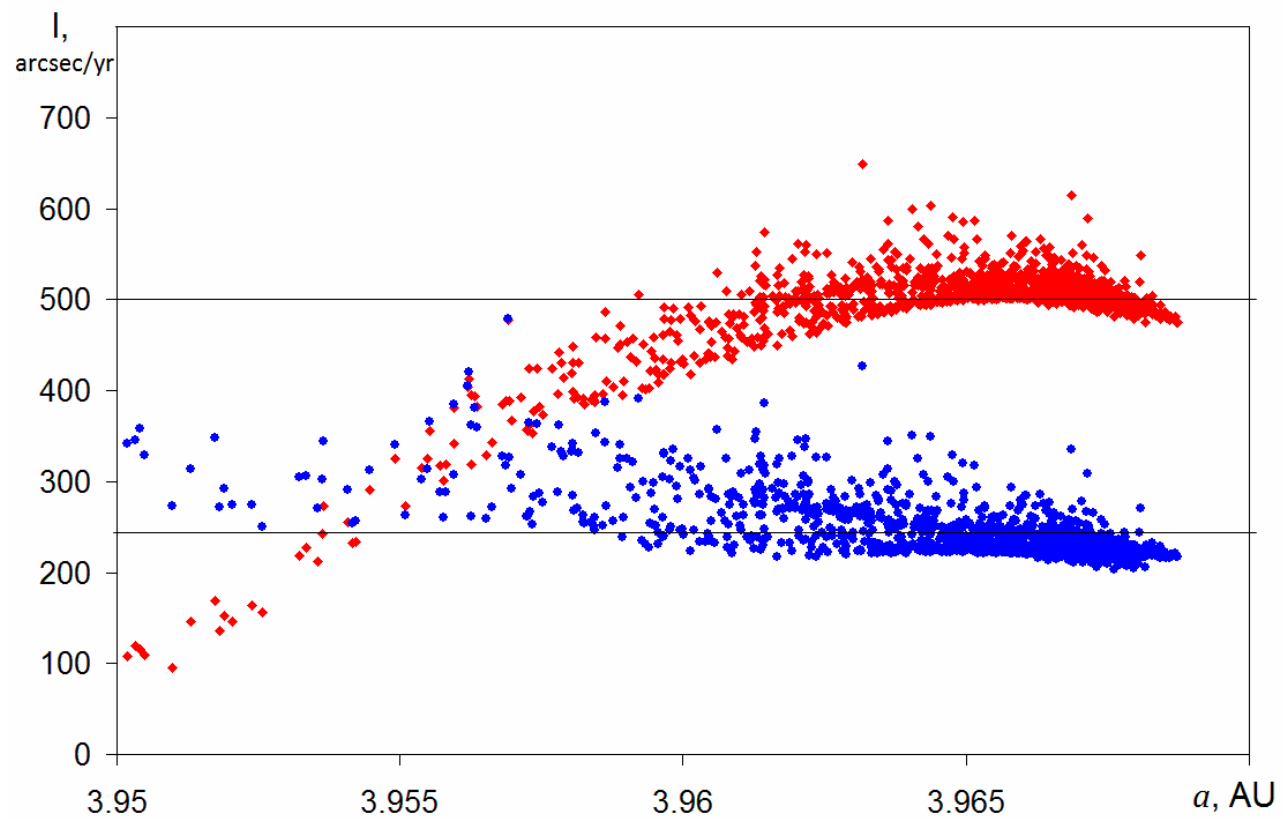


Fig.5. I1 (red) and I2 (blue)

Theoretical background

The value of resonant perihelion rate is (Murray, Dermott, 1999, equation (8.30), table 8.1):

$$\dot{\varpi} = 2n\alpha C_1 + |j_4| C_r e^{|j_4|-2} \overline{\cos \varphi}$$

$$C_r = \frac{1}{2} (m_p / M) n \alpha \left[-2j_1 - \alpha \frac{d}{d\alpha} \right] b_{1/2}^{(j_1)}(\alpha)$$

$$C_1 = \frac{1}{8} (m_p / M) \left[2\alpha \frac{d}{d\alpha} + \alpha^2 \frac{d^2}{d\alpha^2} \right] b_{1/2}^{(0)}(\alpha)$$

The empiric integral:

$$2g - 109296\delta_a + 165.81/e \approx 502.2$$

can be qualitative explained in the theoretical form:

$$g - C_r(\alpha) \frac{0.3 - e}{0.3e} - C_1(\alpha) = \text{const}$$

$$I_2 = 2g - 109296\delta_a + 49 / e^{3/2} \approx 242.2 \Rightarrow ???$$

Conclusions

- In this paper, we demonstrate that the proper elements of asteroids in the 3:2 mean motion resonance with Jupiter are not independent and obtain a relation between them. This relation can be considered as an empirical integral of motion specific of the studied resonance. However, a theoretical explanation of the resulting expression is necessary and may become the subject of future work.
- It is currently unclear whether the coincidence of the coefficient before the λ in our approximation with the Jovian mean motion has any theoretical explanation or whether it is simply accidental. The problem requires further investigation.
- We also pay attention to a group of asteroids that do not match our approximation. All of them have small eccentricities $e < 0.075$. We assume that these asteroids have recently collided or moved under the influence of extremely high non-gravity force. In any case, our numerical integration shows that arbitrary initial orbital elements of the tested particles very rapidly approach the obtained dependences.

Some references

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Thank you!