

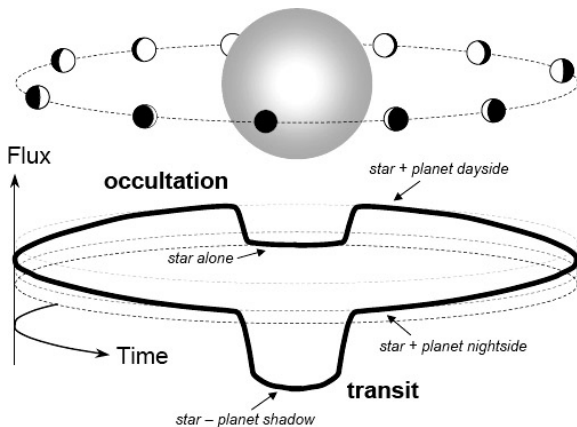
Analytical Methods of Celestial Mechanics 2026
Euler International Mathematical Institute, St. Petersburg, Russia

Exoplanet WASP-76 b secondary eclipse timing deviation possibly indicating the existence of additional unseen components

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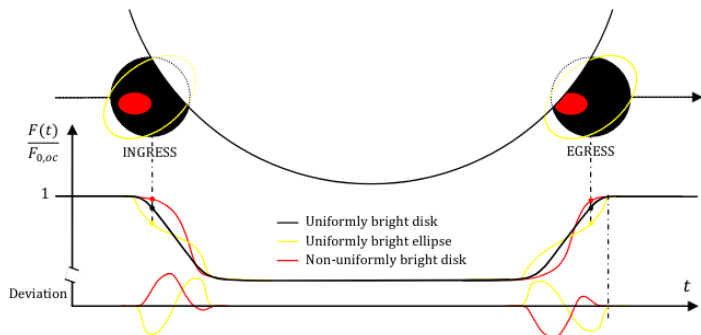
20 August 2026

Secondary eclipses



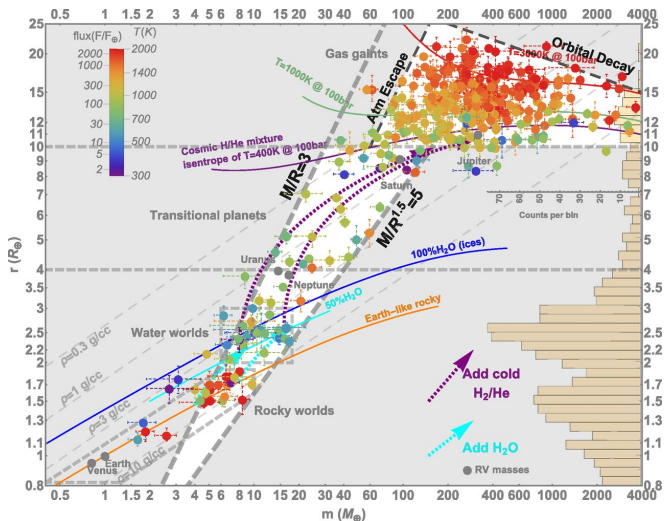
(Giampaolo Piotto and Valerio Nascimbeni, 2015/2016)

Deviation of occultation ingress/egress



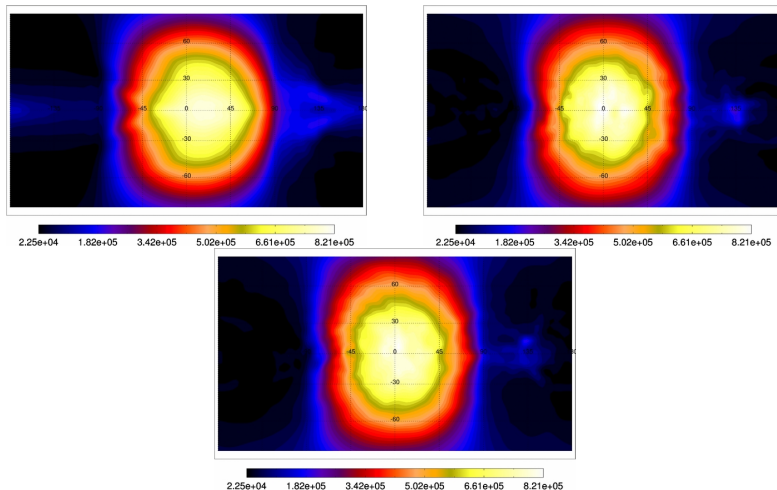
(de Wit, J. et al., 2012, A&A 548, A128)

Hot Jupiters



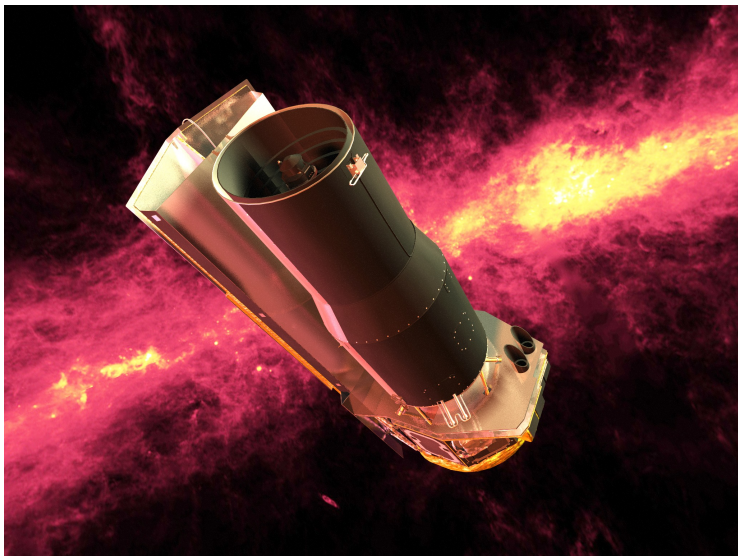
(Zeng, Li et al., 2019, PNAS Vol. 116, Iss. 20, 9723-9728pp.)

Hot spot



Model of IR brightness (W/m^2) HD 209458 b with mag. field 0 G, 3 G and 10 G (Rauscher, Emily & Menou, Kristen, 2013, ApJ Vol. 764, Iss. 1, art. id. 103, 18 pp.)

Spitzer Space Telescope

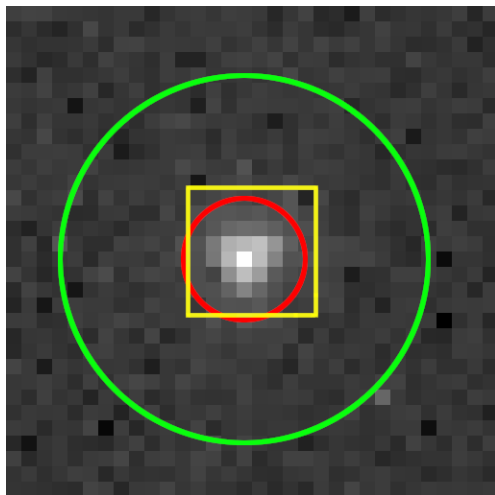


(www.spitzer.caltech.edu/image/sirtf-spitzer-rendered-against-an-infrared-100-micron-sky)

Parameters of exoplanets

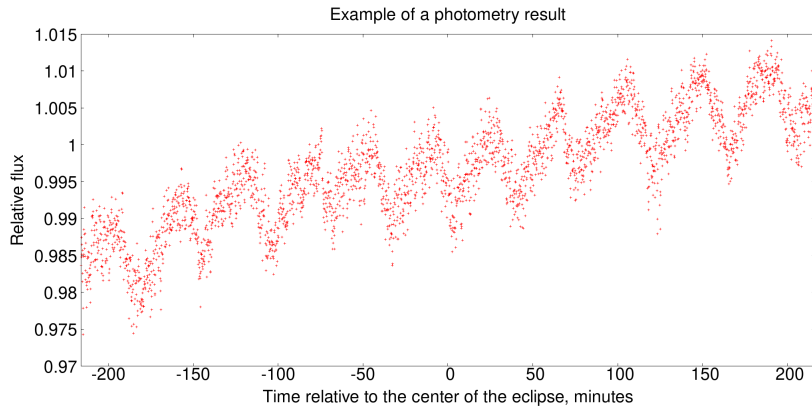
object	R_{star} ($R_{\odot}$)	R_{planet} ($R_{\oplus}$)	a (a.u.)	i ($^{\circ}$)	P (day)
HD 209458 b	1.155	1.359	0.04707	86.710	3.52474859
HD 189733 b	0.756	1.138	0.03100	85.580	2.21857520
GJ 3470 b	0.48	0.346	0.0355	89.13	3.3366524
WASP-12 b	1.657	1.937	0.02320	83.52	1.09142
WASP-43 b	0.6506	1.006	0.01504	82.109	0.813473978
WASP-76 b	1.756	1.854	0.0330	89.623	1.80988198
HAT-P-2 b	1.640	1.157	0.06880	86.72	5.6334754
55 CANCRI e	0.943	0.1673	0.01544	83.59	0.7365474
WASP-80 b	0.571	0.952	0.03427	89.92	3.06785952
WASP-52 b	0.790	1.270	0.02713	85.35	1.7497835

Example of FITS file and data reduction

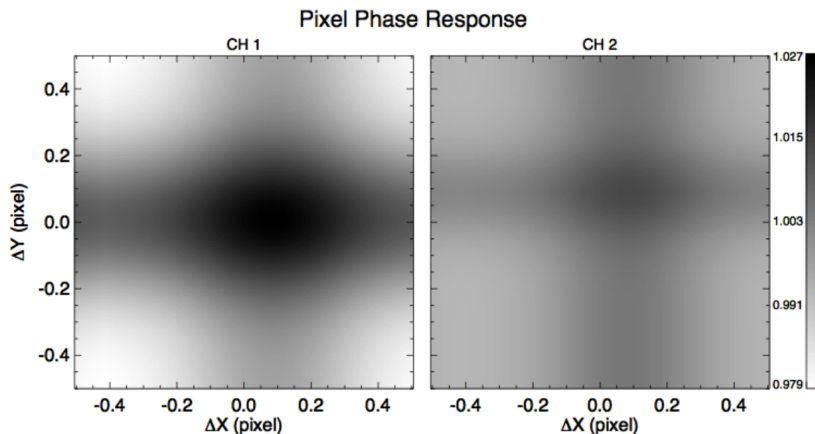


Data reduction was carried out using *astropy* (Astropy Collaboration et al. 2013) and *photutils* (Bradley et al. 2019)

Photometry result



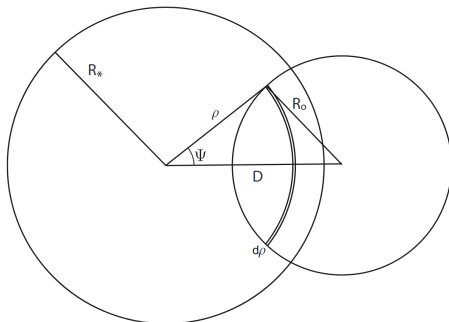
Intrapixel sensitivity variations



IRAC Instrument Handbook, chpt. 4.6, p. 57

Uniformly bright disk model

$$\Delta L(D, R_*, R_o) = \int_0^{R_*} \rho d\rho \int_{-\Psi(\rho, D, R_o)}^{\Psi(\rho, D, R_o)} d\varphi$$



(Abubekrov, M.K. & Gostev, N. Yu., 2013, MNRAS
Vol. 432, Iss. 3, p. 2216-2223)

Secondary eclipse scheme

$$F(t) = F_*(t) + \frac{F_{pl}}{\pi} (\pi - (\Psi(\delta, 1, r) + \Psi(\delta, r, 1)r^2 - Q(\delta, r)/2))$$

$$Q(\delta, r) = \mathcal{Q}((1 - (\delta - r)^2)((\delta + r)^2 - 1))$$

$$\mathcal{Q}(x) = \begin{cases} \sqrt{x} & x \leq 0 \\ 0 & x > 0 \end{cases}$$

$$\Psi(\rho, D, R_o) = \mathcal{A}\left(\frac{\rho^2 + D^2 - R_o^2}{2D\rho}\right)$$

$$\mathcal{A}(x) = \begin{cases} \pi & x < -1 \\ \arccos(x) & -1 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

GP-method

The maximum likelihood method with noise modeling in the form of a stationary Gaussian process. Likelihood function:

$$\mathcal{L}(\boldsymbol{\alpha}, \boldsymbol{\eta}) = \log p(\mathbf{d}|\boldsymbol{\alpha}, \boldsymbol{\eta}) + \frac{N}{2} \log(2\pi) = -\frac{1}{2} \log \det \mathbf{V} - \frac{1}{2} (\mathbf{d} - \boldsymbol{\mu})^T \mathbf{V}^{-1} (\mathbf{d} - \boldsymbol{\mu}),$$

where $\mathbf{d}$ – series of observations;

$\boldsymbol{\mu}$ – the model of the eclipsed light curve, which depends on the parameters $\boldsymbol{\alpha}$;

$\mathbf{V}$ – the covariance matrix, which depends on the parameters $\boldsymbol{\eta}$.

Modified likelihood function from the work (Baluev R.V., 2009, MNRAS, 393, 969):

$$\tilde{\mathcal{L}}(\boldsymbol{\alpha}, \boldsymbol{\eta}) = -\frac{1}{2} \log \det \mathbf{V} - \frac{1}{2\gamma} (\mathbf{d} - \boldsymbol{\mu})^T \mathbf{V}^{-1} (\mathbf{d} - \boldsymbol{\mu}), \gamma = 1 - \frac{\dim \boldsymbol{\alpha}}{N}.$$

Description of the model

Correlation function:

$$V_{ij} = k(\Delta t_{ij} | \boldsymbol{\eta}) = \sigma_0^2 \delta_{ij} + \sigma_1^2 \exp\left(-\frac{\Delta t_{ij}^2}{2\tau_1^2}\right) + \sigma_2^2 \exp\left(-\frac{\Delta t_{ij}^2}{2\tau_2^2}\right), \boldsymbol{\eta} = \{\sigma_0, \sigma_1, \tau_1, \sigma_2, \tau_2\}.$$

The eclipsed light curve model:

$$\mu_i = P_3(t_i) + F_{pl}(1 - \Delta L(D_i, R_*, R_o)) + f(x_i, y_i),$$

where P_3 – a third degree polynomial;

ΔL – the fraction of a planet obscured by a star;

f – a systematic error caused by the intrapixel sensitivity variations.

The model is described in more detail in the article (Fedotov & Baluev, 2026, A. Bull. V. 81 P. 340-359).

Description of the model

An approximation in the form of a Fourier series was used. Example with 20 harmonics:

$$\begin{aligned} f(x_i, y_i) = & a_1 \sin(2\pi x_i) + b_1 \cos(2\pi x_i) + a_2 \sin(2\pi y_i) + b_2 \cos(2\pi y_i) \\ & + a_3 \sin(2\pi(x_i + y_i)) + b_3 \cos(2\pi(x_i + y_i)) \\ & + a_4 \sin(2\pi(x_i - y_i)) + b_4 \cos(2\pi(x_i - y_i)) \\ & + a_5 \sin(4\pi x_i) + b_5 \cos(4\pi x_i) + a_6 \sin(4\pi y_i) + b_6 \cos(4\pi y_i) \\ & + a_7 \sin(2\pi(x_i + 2y_i)) + b_7 \cos(2\pi(x_i + 2y_i)) \\ & + a_8 \sin(2\pi(x_i - 2y_i)) + b_8 \cos(2\pi(x_i - 2y_i)) \\ & + a_9 \sin(2\pi(2x_i + y_i)) + b_9 \cos(2\pi(2x_i + y_i)) \\ & + a_{10} \sin(2\pi(2x_i - y_i)) + b_{10} \cos(2\pi(2x_i - y_i)) \end{aligned}$$

Modeling results

	The offset of the eclipse center in the radii of the planet	Effective temperature of the planet (K)
HD209458b	-0.036 ± 0.057	1592 ± 59
HD189733b	0.021 ± 0.044	1319 ± 58
GJ3470b ^{1,2}	9.32 ± 2.35	740^{+180}_{-740}
WASP-12b	0.033 ± 0.098	2600 ± 280
WASP-43b	-0.063 ± 0.076	1936 ± 89
WASP-76b	-0.217 ± 0.051	2560 ± 300
HAT-P-2b ¹	0.039 ± 0.193	2210 ± 390
55 Cancri e ²	3.428 ± 1.755	2500^{+1700}_{-1100}
WASP-80b	-0.126 ± 0.184	1019 ± 74
WASP-52b	0.003 ± 0.198	1390 ± 100

1 – planets with high eccentricity, 2 – planets with low accuracy in detecting secondary eclipses. Planets highlighted in red may have issues with the accuracy of their data, while those highlighted in green show a statistically significant offset.

Results WASP-76 b

The offset of the eclipse center can be explained by the nonuniform brightness or the eccentricity of the orbit.

If the offset is explained by eccentricity, we obtain

$$e \cos(\omega) = -0.00146 \pm 0.00034.$$

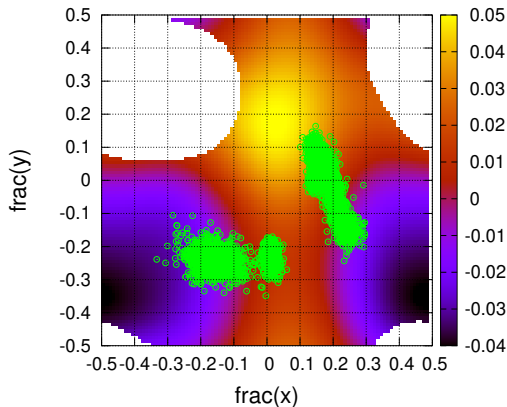
This result does not contradict the result of other authors (Fu Guangwei et al. 2021)

$$e \cos(\omega) = 0.0036^{+0.013}_{-0.0086},$$

obtained based on observations of secondary eclipses in several wavelength ranges.

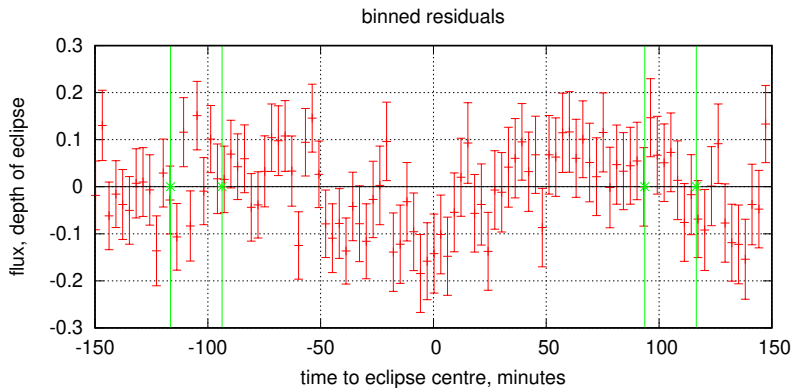
Existence of a non-zero eccentricity should indicate perturbations from unknown objects in the system.

Systematic error depending on the center coordinates



A 2D map built based on the analysis of data of WASP-76 b.

Result of binning the residuals



The results of binning the residuals of the best fitting model for the exoplanet WASP-76 b, with a step of 3 minutes.

- Exoplanet WASP-76 b secondary eclipse timing deviation possibly indicating the existence of additional unseen components

(Fedotov Andrei Alekseevich, coauthor Baluev Roman Vladimirovich)

Conclusions:

- For many exoplanets, a correlated noise model based on Gaussian processes was constructed for Spitzer telescope data, depending on the coordinates of the object's center on the telescope's detector array.
- The presence of remaining red noise has been confirmed after taking into account the intrapixel sensitivity variations.
- A constraint has been obtained on the systematic offset of the eclipse moment for many exoplanets. The offset for reliable cases without noticeable eccentricity turned out to be -0.010 ± 0.029 of the planet's radius.
- The offset is most noticeable for WASP-76b. It is -0.217 ± 0.051 of the planet's radius and corresponds to the value of the eccentricity $e \cos(\omega) = -0.00146 \pm 0.00034$. Since the sign of the offset is negative, this is probably not a hot spot, but an effect of eccentricity, which may be caused by a perturbation from another object.

Bibliography

- Zeng, Li et al., 2019, PNAS, 'Growth model interpretation of planet size distribution'. arXiv:[1906.04253](#)
- Giampaolo Piotto and Valerio Nascimbeni, 2015/2016, UNIVERSITA DEGLI STUDI DI PADOVA, 'Identificazione e caratterizzazione di sistemi planetari nei campi di vista di PLATO'. Corso di Laurea in Astronomia
- de Wit, J. et al., 2012, A&A, 'Towards consistent mapping of distant worlds: secondary-eclipse scanning of the exoplanet HD 189733b'. arXiv:[1202.3829](#)
- Rauscher, Emily and Menou, Kristen, 2013, ApJ, Volume 764, Issue 1, article id. 103, 18 pp., 'Three-dimensional Atmospheric Circulation Models of HD 189733b and HD 209458b with Consistent Magnetic Drag and Ohmic Dissipation'. arXiv:[1208.2274](#)
- Abubekrov, M. K. and Gostev, N. Yu., 2013, MNRAS, 432.3, 2216–2223, 'A universal approach to the calculation of the transit light curves'. arXiv:[1304.1378](#)
- Baluev, R. V., 2009, Astronomy and Computing, 25, 221–229, 'PlanetPack3: a radial-velocity and transit analysis tool for exoplanets'. arXiv:[1809.07052](#)
- Fu Guangwei et al., 2021, The Astronomical Journal 162.3, 18 'The Hubble PanCET Program: Transit and Eclipse Spectroscopy of the Strongly Irradiated Giant Exoplanet WASP-76b'. arXiv:[2005.02568](#)
- A.A. Fedotov, R.V. Baluev, 2026, Astrophysical Bulletin, V. 81 P. 340–359 'The effect of Spitzer/IRAC systematic noises on secondary eclipses of the exoplanet HD209458 b'

Thank you for your attention!