Dynamics of entering and leaving the quasi-satellite regime among the known Earth co-orbitals

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Abstract. A quasi-satellite is a small body that orbits the Sun while remaining near a planet for an extended period without being gravitationally bound to it. Quasi-satellites are an important part of the population of co-orbitals of planets in the Solar System. For Earth, quasi-satellites can be described as a subclass of co-orbitals that is better identified than other subclasses. The first quasi-satellite of Earth was revealed in 2004 and subsequently received the name Cardea. Currently, seven asteroids are more or less unanimously classified by specialists as Earth's quasi-satellites. Since the quasi-satellite motion of all known objects is temporary, understanding the patterns of its transformation is of great importance. To study these patterns, we propose a semi-analytical model that describes the secular evolution of asteroid motion in the 1:1 mean-motion resonance with Earth. As an example, we consider the repeated transitions of asteroid (164207) Cardea into and out of the quasi-satellite regime.

Introduction

For near-Earth asteroids, the 1:1 mean-motion resonance (MMR) manifests itself through the oscillatory behaviour of the resonant phase $\varphi = \lambda - \lambda_3$, where λ is the asteroid's mean longitude and λ_3 is Earth's mean longitude. In general, this resonance admits several distinct regimes: the quasi-satellite (QS), Trojan (T), and horseshoe (HS) regimes, as well as various compound states (e.g., QS+HS). These regimes differ in the value of the resonant phase around which they librate and in the libration amplitude [1]. In the QS regime, in particular, the resonant phase φ librates around a value close to $0°$.

Studying a system with several coexisting resonant regimes requires analysing not only the transitions from non-resonant to resonant motion and vice versa, but also the transitions between different resonant modes. Such mutual transitions of

near-Earth asteroids between the QS regime and other regimes allowed by the 1:1 MMR have been examined, for instance, in [2, 3].

To study the secular evolution of quasi-satellite motions and to identify possible transformation scenarios, [4] and [5] used an analysis of approximate integrals of motion - adiabatic invariants (AI). The systematic use of this approach for investigating resonant dynamics in Celestial Mechanics was pioneered in [6]. How this method should be adapted to situations with coexisting resonant regimes is discussed in [7].

The studies [4] and [5] were mainly methodological in scope, focusing on the general features of quasi-satellite motion. Given the considerable number of known quasi-satellite asteroids, it is relevant to apply the adiabatic approximation systematically - to better understand the dynamics of both individual asteroids and the population as a whole. As a first step, we chose to examine the population of Earth's quasi-satellites.

Our preliminary estimations have shown that a realistic dynamical model cannot rely solely on Earth's gravitational attraction to describe the secular evolution of the heliocentric motion of its co-orbitals. Since [4] and [5] were based on the restricted three-body problem, we had to extend their model to include perturbations from five planets: Venus, Earth, Mars, Jupiter, and Saturn.

1. Semi-analytical model for the transformation of motion in the 1:1 MMR

In our semi-analytical model, we treated the planets as point masses moving on circular coplanar orbits. The equations of motion were initially written in Delaunay variables. We then applied a canonical transformation that made the resonant phase φ one of the phase variables.

The Hamiltonian of the problem was averaged over the fast orbital motions of the planets and the asteroid. The resulting intermediate system was split into a subsystem describing the evolution of φ and its conjugate momentum Φ, and a subsystem describing the secular evolution of the eccentricity e and the argument of perihelion ω. This separation was justified by the different characteristic rates of change of these variables:

$$\dot{\varphi} \sim \dot{\Phi} \gg \dot{e} \sim \dot{\omega}.$$

For fixed slow variables e and ω, the subsystem for the resonant phase is a Hamiltonian system with one degree of freedom. The action variable of this 1DOF Hamiltonian is an adiabatic invariant of the problem.

A further averaging, now performed along the solutions of the subsystem for the resonant phase, leads to evolutionary equations for the slow variables. Then, on the (e, ω) plane, one can construct regions such that when the solutions of the evolutionary equations reach the boundaries of these regions, transitions between resonant regimes occur (either in a deterministic or a quasi-probabilistic manner).

2. Transitions between resonant regimes for asteroid (164207) Cardea

Figure 1 shows the behaviour of the phase trajectories of the evolutionary equations for parameter values corresponding to the case of asteroid (164207) Cardea. The red lines show the evolution of the osculating elements e and ω during the secular evolution of the asteroid's orbit, obtained by numerical integration. The left half of the figure presents the results of the integration with initial data taken from the NASA/JPL Small?Body Database (SBDB). The right half shows the evolution of a clone of the asteroid, which differs in the initial value of the mean anomaly.

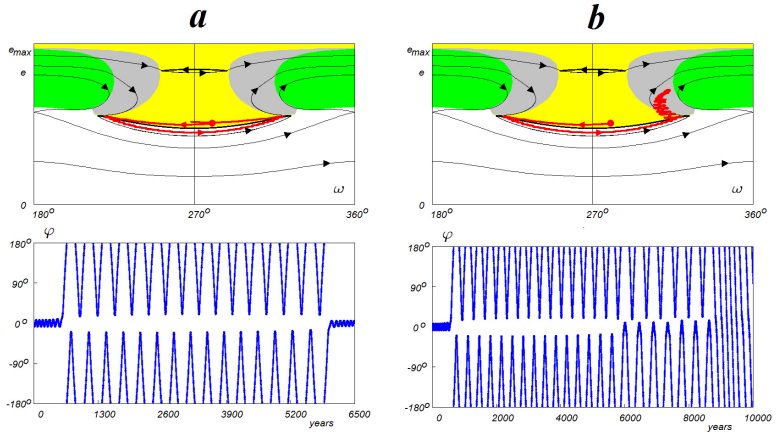


FIGURE 1. Presumed transitions between resonant regimes for asteroid (164207) Cardea. The upper part shows the phase portrait in the plane of slow variables (yellow region: coexistence of QS and HS regimes; grey region: compound QS+HS regime; white region: HS regime; green region: non-resonant(passing) regime). Panel (a) corresponds to the nominal values of the osculating variables; panel (b) corresponds to a clone differing in the mean anomaly (with a difference of $1.3 \cdot 10^{-4}$ degrees).

From the diagram, it can be seen that the asteroid is currently located in the region of coexistence of the QS and HS regimes (highlighted in yellow). The phase point characterising the dynamical state of the asteroid will then shift to the left until it reaches the left boundary of this region. An analysis of the behaviour of the trajectories of the evolutionary equations shows that the vicinity of the boundary can only be left in the HS regime. Thus, the first regime transition for Cardea should be deterministic. The evolution in the HS regime will bring the phase point to the right boundary of the coexistence region. Here, the transition is of a quasi-probabilistic nature: the asteroid may either return to the QS regime (Fig. 1a) or

enter the compound QS+HS regime (Fig. 1b). The transition probabilities can be calculated using the formulas given in [7]:

$$P_{HS\to QS} \approx 0.94461, \quad P_{HS\to QS+HS} \approx 0.05539.$$

Conclusion

The main result of our study is that a number of Earth's quasi-satellites (Cardea, Kamo'oalewa) and some of its other co-orbitals ($2015\mathrm{SO}_2$, $2016\mathrm{CO}_{246}$, $2020\mathrm{PN}_1$) can be regarded as belonging to the population of regularly transient asteroids. During their secular evolution, these asteroids alternate between the QS and HS regimes, occasionally interrupted by transitions into the compound QS+HS regime. With the transition patterns established, we plan to obtain estimates of the distribution of regularly transient asteroids among the different regimes.

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