# On the solution of the special BVP related to a full set of geodetic and gravimetric data

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**Abstract.** The work is devoted to the problem of determining the anomalous gravitational potential on the earth's surface in conditions of the simultaneous availability of data on the spatial position of points, their height above sea level and the measured force of gravity. A solution to the boundary value problem is proposed based on gravity anomalies of a special type. The necessary mathematical expressions have been obtained. Computational experiments were performed on model data.

## Introduction

There are two main types of gravity reductions: "free-air" reductions and topographic reductions. The former are used to solve boundary value problems in physical geodesy, while the latter are used to improve the accuracy of calculations and facilitate interpolation using a gravimetric map.

For a known boundary surface, to determine the anomalous potential, L.P. Pellinen [1] proposed to introduce anomalies of a special type that combine the advantages of known "free air" anomalies (gravity anomalies – used with a known potential difference, gravity disturbance – with a known boundary surface) and take into account the influence of topographic masses on the calculation point. He formulated a boundary condition in the spherical approximation and proposed a solution based on the method of expansion in powers of a small parameter of M.S. Molodensky [3]. These anomalies are of particular interest because it is now possible to simultaneously obtain geodetic, leveling and gravimetric determinations for one point, leading to different in gravity reductions.

## 1. Analytical solution

First, a solution to the boundary value problem was obtained in integral form for a spherical reference surface. In addition, algorithms for taking into account the

influence of the "zero-zone" have been developed for the numerical implementation of the obtained formula, taking into account the singularity at the evaluation point.

To ensure the possibility of increasing the accuracy of calculations, an integral equation is obtained based on the density of a simple layer, taking into account the flattened of the reference surface. A necessary boundary condition is proposed. A connection was obtained between gravity anomalies in special reduction and other types of anomalies.

The properties of the obtained solutions are described.

## 2. Computational experiments

To test the obtained solutions, Earth models were created: sources of normal and anomalous fields were selected, the physical surface was defined, reference values of the determined quantities and measurement results were generated. Two types of calculations were performed:

- in the first case, the boundary surface and the values of gravity anomalies are considered known, and the anomalous potential is subject to determination;
- in the second case, the boundary surface is also known, the values of the gravity anomaly are given, and it is necessary to calculate the anomalous potential.

Since it is recommended to move from gravity anomalies to gravity disturbance ones to refine the solution of the boundary value problem [2], in both experiments the results of calculations based on anomalies in a special reduction and gravity disturbance were compared.

## Conclusion

The spherical solution has a number of interesting properties: the integration kernel does not contain transcendental functions, the first-degree harmonic is preserved, the considered type of anomalies in this form is endowed with the meaning of the density of a simple layer distributed over the surface under study. The algorithms for taking into account the influence of the "zero-zone" seem to be quite simple.

The solution in the form of an integral equation over the surface has a simpler structure than a similar representation for other types of anomalies. With numerical integration, the rate of convergence of its solution is several times higher than for gravity disturbance. There is a tendency towards a uniform distribution of calculation errors over the entire integration surface, in contrast to gravity disturbance, which strongly correlate with the magnitude of the anomalous potential itself.

A solution based on special type anomalies will be convenient for calculating the gravimetric height anomaly for satellite leveling, since the calculation results will contain only the methodological error in performing the numerical integration, which can be studied separately.

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