

A numerical survey of von Zeipel–Lidov–Kozai resonances and their coupling with mean-motion resonances in the trans-Neptunian region and the inner Solar System

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Abstract. We present two catalog-wide numerical surveys of the von Zeipel–Lidov–Kozai (ZLK) resonance and its coupling with mean-motion resonances (MMRs), one in the trans-Neptunian region ($a > 30$ au) and one inside Jupiter’s orbit ($a < 4.9$ au). Beyond Neptune every one of the 81 ZLK-resonant objects is also trapped in a two-body MMR with Neptune, while inside Jupiter’s orbit only 11 of the 201 ZLK-resonant asteroids are in an MMR. The coupling is thus strong and universal beyond Neptune, but weak inside Jupiter’s orbit, where the ZLK resonance is an overwhelmingly planet-crossing phenomenon that extends below the classical critical inclination.

Introduction

The von Zeipel–Lidov–Kozai (ZLK) resonance, first found by von Zeipel (1910) and later developed by Lidov (1962) and Kozai (1962), produces a libration of the argument of pericenter ω with coupled anti-correlated oscillations of the eccentricity e and inclination i . In the quadrupole test-particle limit the libration is possible above a critical inclination $\arccos \sqrt{3/5} \approx 39.2^\circ$. This value holds only in the $\alpha \rightarrow 0$ limit ($\alpha = a/a_p$, the ratio of the two semi-major axes) and grows with α (Kozai 1962; Vashkoviak 1981). Following Morbidelli (2002); Shevchenko (2017); Sidorenko (2018) we treat the ZLK effect as a nonlinear resonance with resonant angle $\sigma \equiv \omega$; a full account of the theory is given by Shevchenko (2017).

In the outer Solar System the ZLK resonance is closely tied to MMRs. Gallardo (2006) showed that the locking of the semi-major axis inside an MMR allows ZLK libration below the non-resonant critical inclination, and Lei et al. (2022) showed that the libration centers can then shift away from 90° and 270° . Inside Jupiter’s orbit the picture is more fragmented: the libration centers shift to 0° and 180° in the low-inclination, planet-crossing regime (Michel and Thomas 1996;

Gronchi and Milani 1999), where Carruba et al. (2026) recently identified ZLK-resonant co-orbitals of the terrestrial planets. A catalog-wide census of the ZLK resonance had not been carried out in either region. We provide such a census for both and test whether the tight MMR–ZLK coupling seen for trans-Neptunian objects (TNOs) also holds inside Jupiter’s orbit. The full surveys are presented in Smirnov and Milić Žitnik (2026a) and Smirnov and Milić Žitnik (2026b).

1 Methods

Both surveys use the **resonances** package (Smirnov 2023), which wraps **rebound** (Rein and Liu 2012), in a full N-body model with the Sun, all planets, and Pluto. For each object the angle $\sigma \equiv \omega$ was classified as resonant (a clean libration over at least one full cycle), possible (temporarily captured or near the separatrix), or non-resonant, and inside Jupiter’s orbit the librators were further split into regular and irregular ones. The two integration setups differ because the ZLK timescales do. For the 1037 TNOs with $a > 30$ au we integrated for 10 Myr and re-integrated the ambiguous objects in stages up to 300 Myr. For the 863 471 asteroids with $a < 4.9$ au we used a 100 kyr automated phase and a 500 kyr manual phase; because many of these objects are highly eccentric, highly inclined, or planet-crossing, only the **ias15** integrator (Rein and Spiegel 2015) gave stable results for the whole sample. Every resonant object was then checked for concurrent membership in two-body and three-body MMRs.

2 Results

From the 1037 TNOs, 81 (7.8%) are in the ZLK resonance and 23 are transient. The main result is that all 81 ZLK-resonant TNOs are simultaneously trapped in a two-body MMR with Neptune, a direct empirical confirmation of the predicted coupling in this region. The most populated resonances are 2N-3 (58 objects), 4N-7 (15), 1N-2 (9), and 3N-5 (7), which together hold about 86% of the ZLK-resonant TNOs. The libration centers cluster near 90° and 270° , except in 1N-2, where they are shifted to asymmetric values, as expected for 1:N resonances. Among individual objects, the retrograde TNO (585899) 2020 HM98 is temporarily in the resonance 2N+9, and the most distant, (652920) 2014 GR53 at $a = 202.8$ au, is in 1N-18.

From the 863 471 asteroids, 201 are confirmed ZLK-resonant (163 regular and 38 irregular), less than 0.02% of the sample. The population is overwhelmingly planet-crossing: 144 are near-Earth asteroids ($q < 1.3$ au) and 28 are Mars-crossers, while the main belt is almost empty. In sharp contrast with the TNOs, only 11 of the 201 (about 5%) are also in a two-body or three-body MMR, and the surviving pairs concentrate in the 2J-1 resonance with Jupiter, so the two resonances appear decoupled here. About one-third of the librators (62 objects) have inclinations below 39.2° , and 24 lie above the quadrupole threshold $c_1 = (1 - e^2) \cos^2 i = 3/5$, which the classical test-particle theory forbids. This is consistent with the shifted equilibria inside MMRs (Gallardo 2006; Lei et al. 2022) and with the planet-crossing centers at $\omega = 0^\circ$ and 180° (Michel and Thomas 1996; Gronchi and Milani

1999; Carruba et al. 2026), and part of it with the higher α -corrected threshold. The four regular 2J-1 librators fall on the highly inclined island of the two-island structure of Chrenko et al. (2015).

Conclusions

Beyond Neptune the ZLK resonance and the MMR network are tightly coupled: every ZLK-resonant TNO is locked in a two-body MMR with Neptune, and the locking of the semi-major axis creates the conditions for the ZLK libration. Inside Jupiter's orbit this coupling is almost absent. Fewer than 6% of the ZLK-resonant asteroids are in an MMR, the surviving pairs concentrate in the 2J-1 resonance, and the ZLK resonance is instead a planet-crossing phenomenon that reaches below the classical critical inclination. We attribute the contrast to MMR overlap and chaos in the main belt, to eccentricity pumping onto planet-crossing orbits, and to competition with strong secular resonances.

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