

A numerical survey of **von Zeipel-Lidov-Kozai resonances** and their coupling with mean-motion resonances in the trans-Neptunian region and the inner Solar System

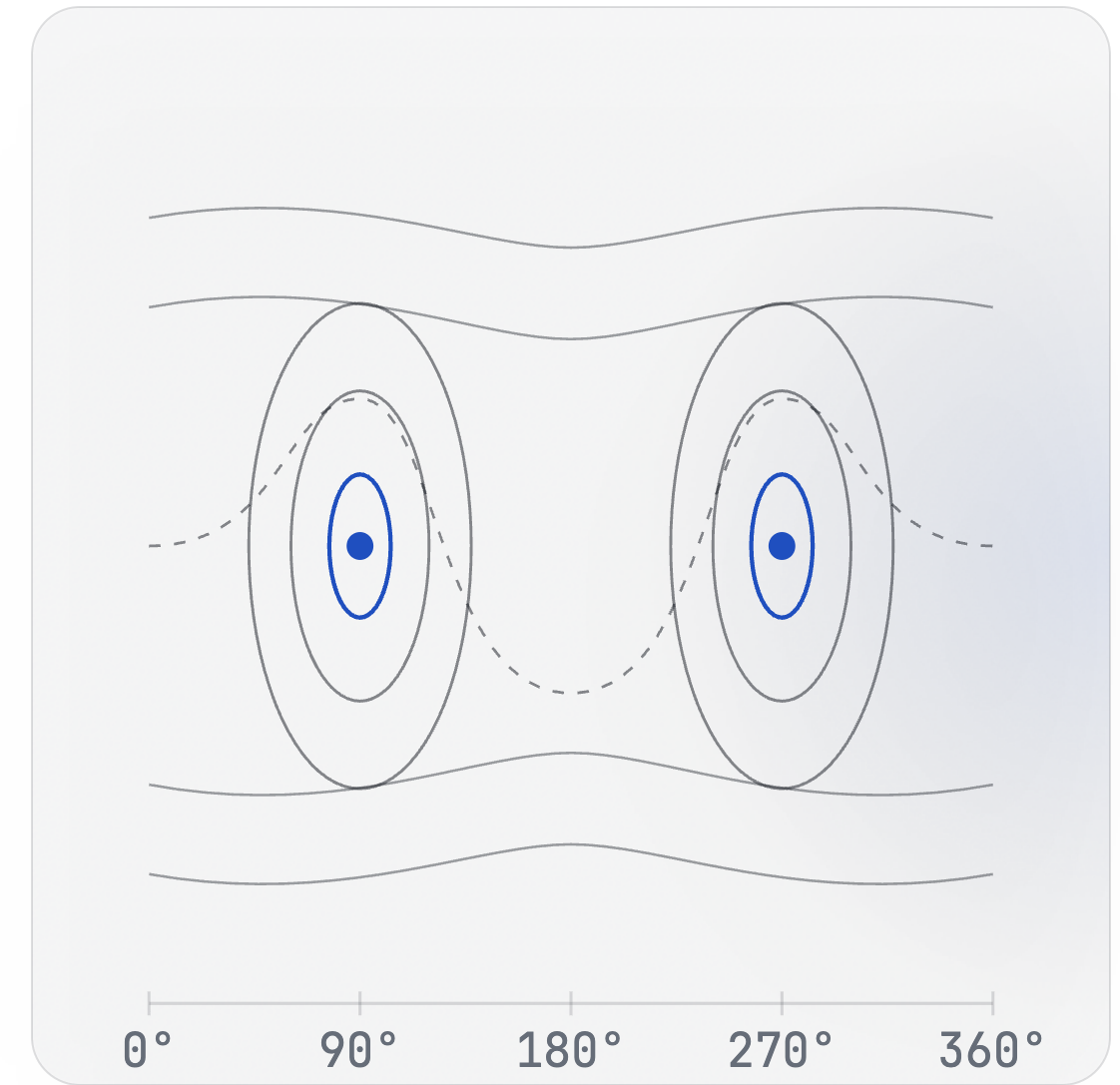
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The von Zeipel-Lidov-Kozai mechanism is a dynamical phenomenon that occurs in hierarchical three-body systems, where the gravitational influence of a distant perturber leads to significant oscillations in the **eccentricity** and **inclination** of an orbiting body.

THROUGHOUT THIS TALK

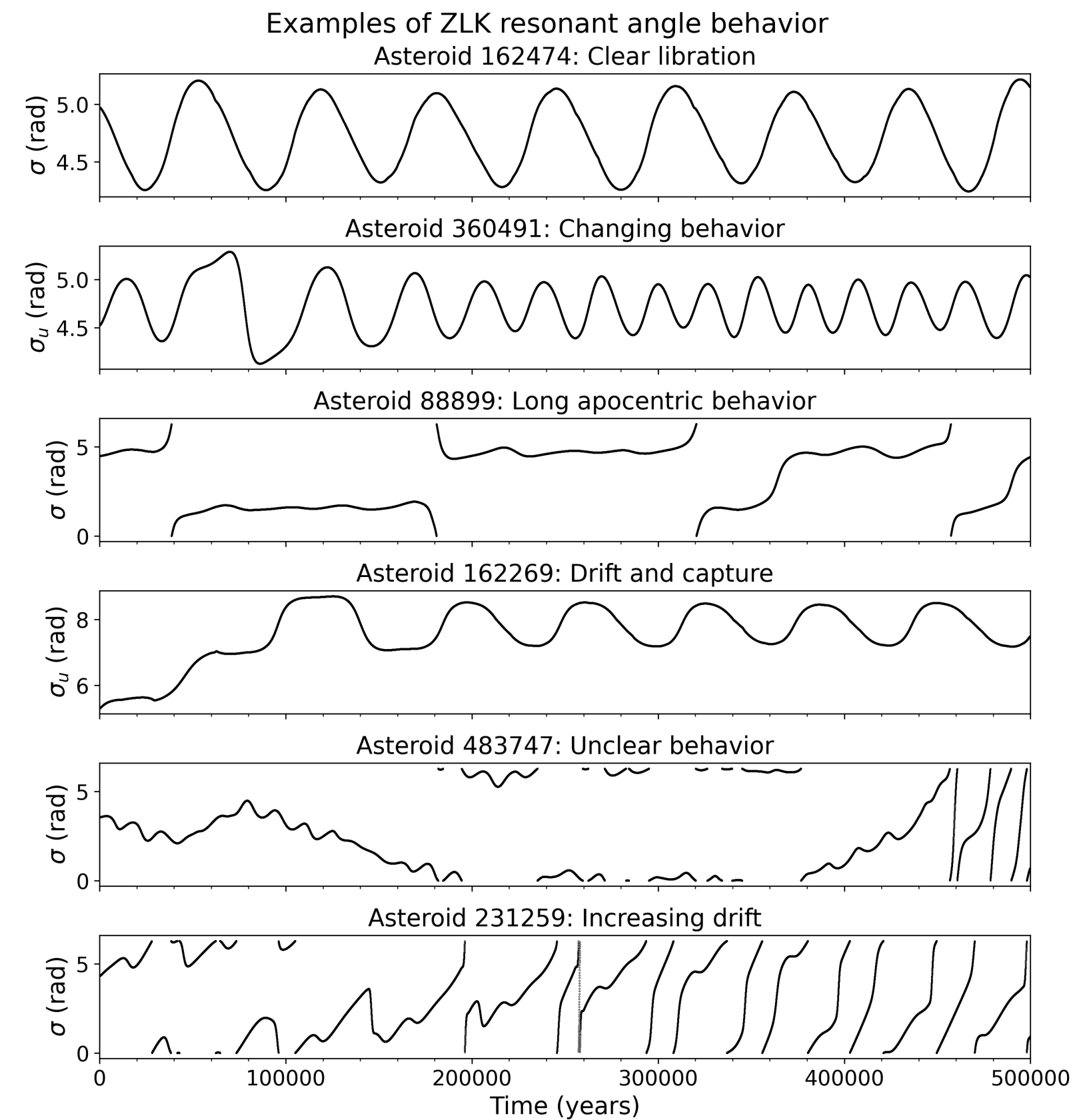
$$\sigma \equiv \omega$$

the resonant angle is the
argument of pericentre

Libration is rarely textbook-clean.

Clear libration, changing behaviour, long apocentric libration, drift and capture, unclear behaviour, increasing drift — all over 500 kyr.

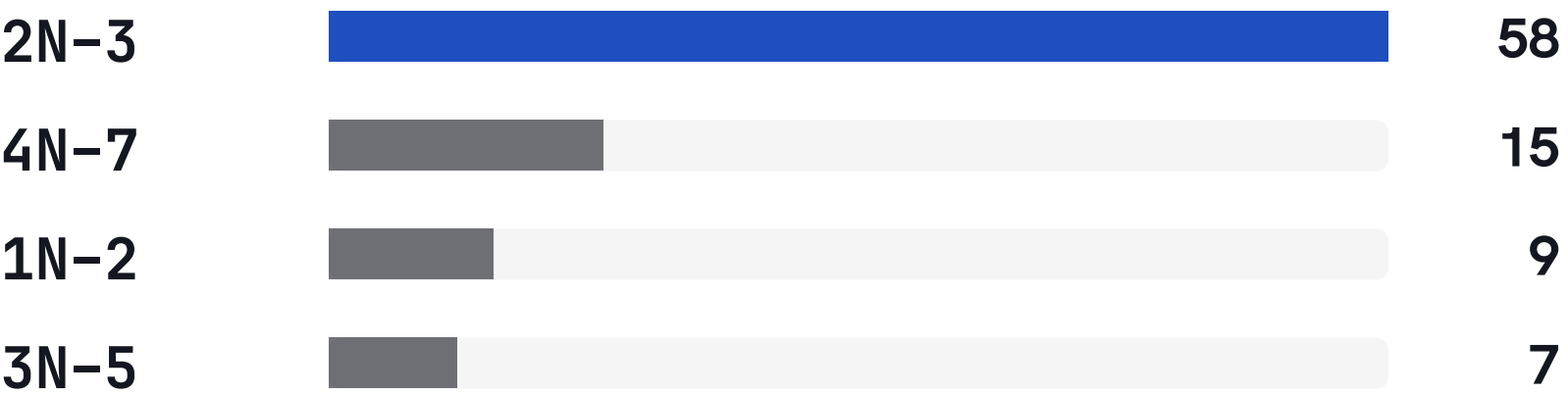
Smirnov & Žitnik, 2026a, *Icarus*



81 of 1037 numbered TNOs
are in ZLKR — about 7.8%

100% of those 81
are also in mean-motion resonances

Most populated MMRs



INTERESTING CASE

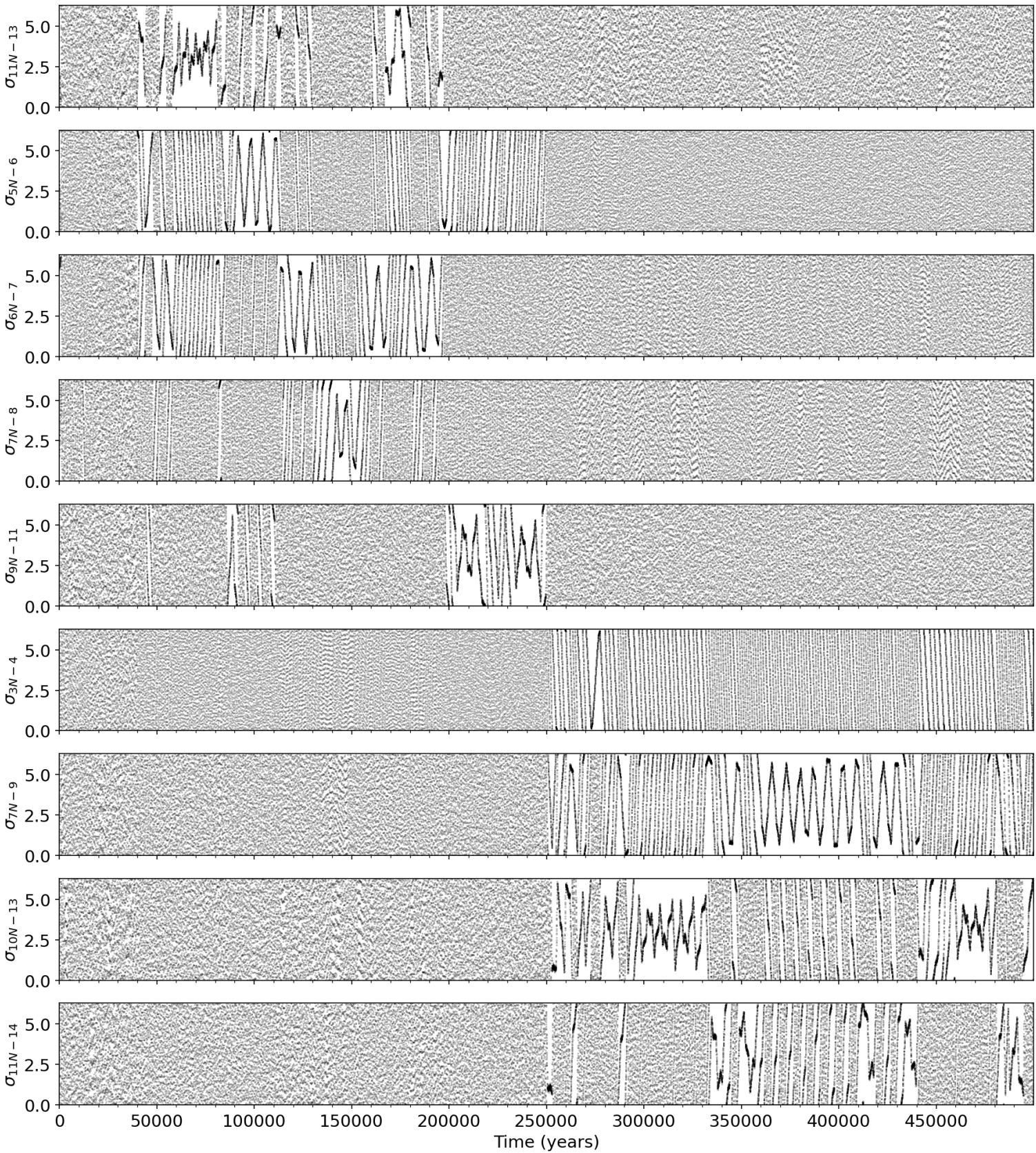
(652920) 2014 GR₅₃

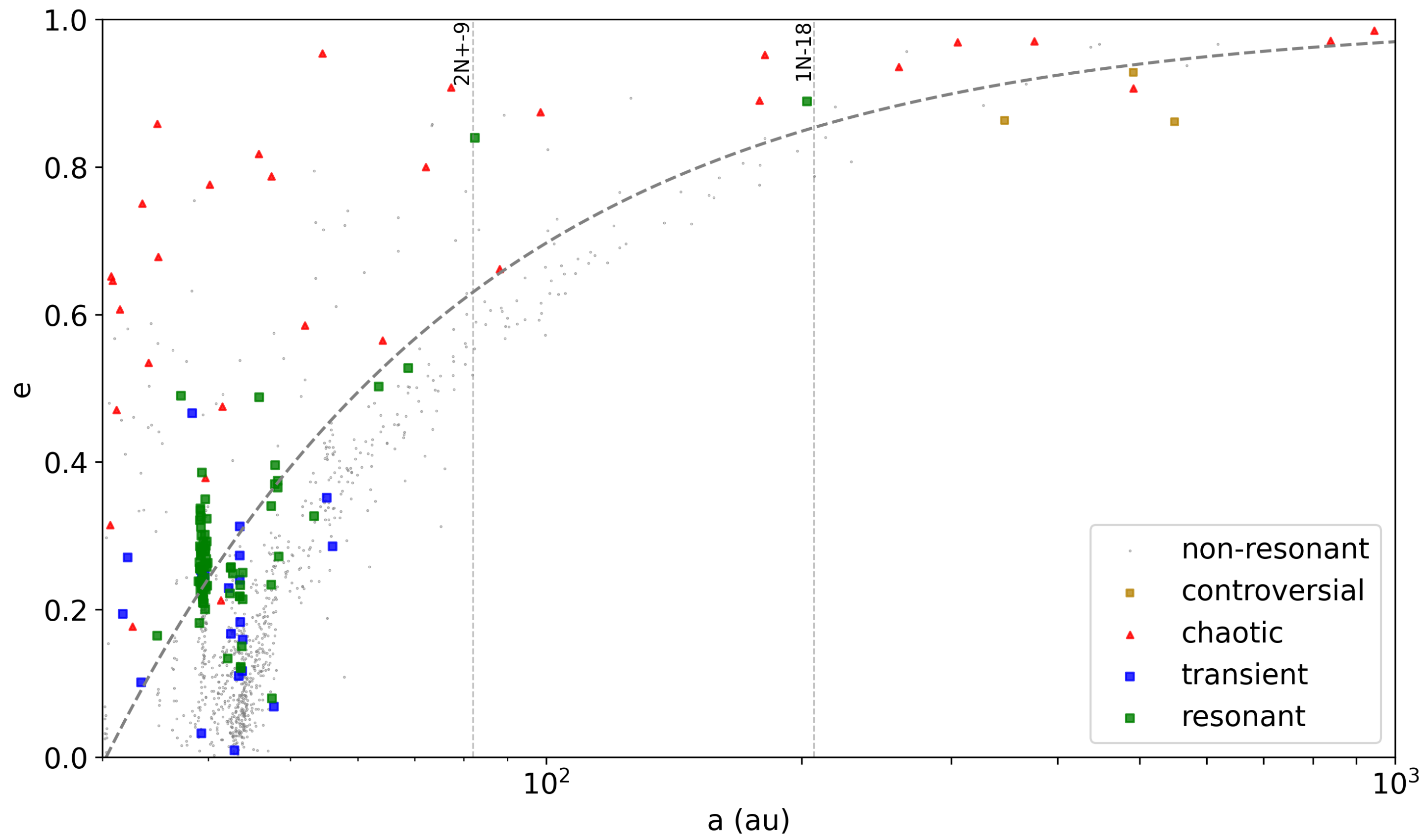
A distant TNO, a ≈ 202.8 au, trapped in the **1N-18** MMR *and* in ZLKR.

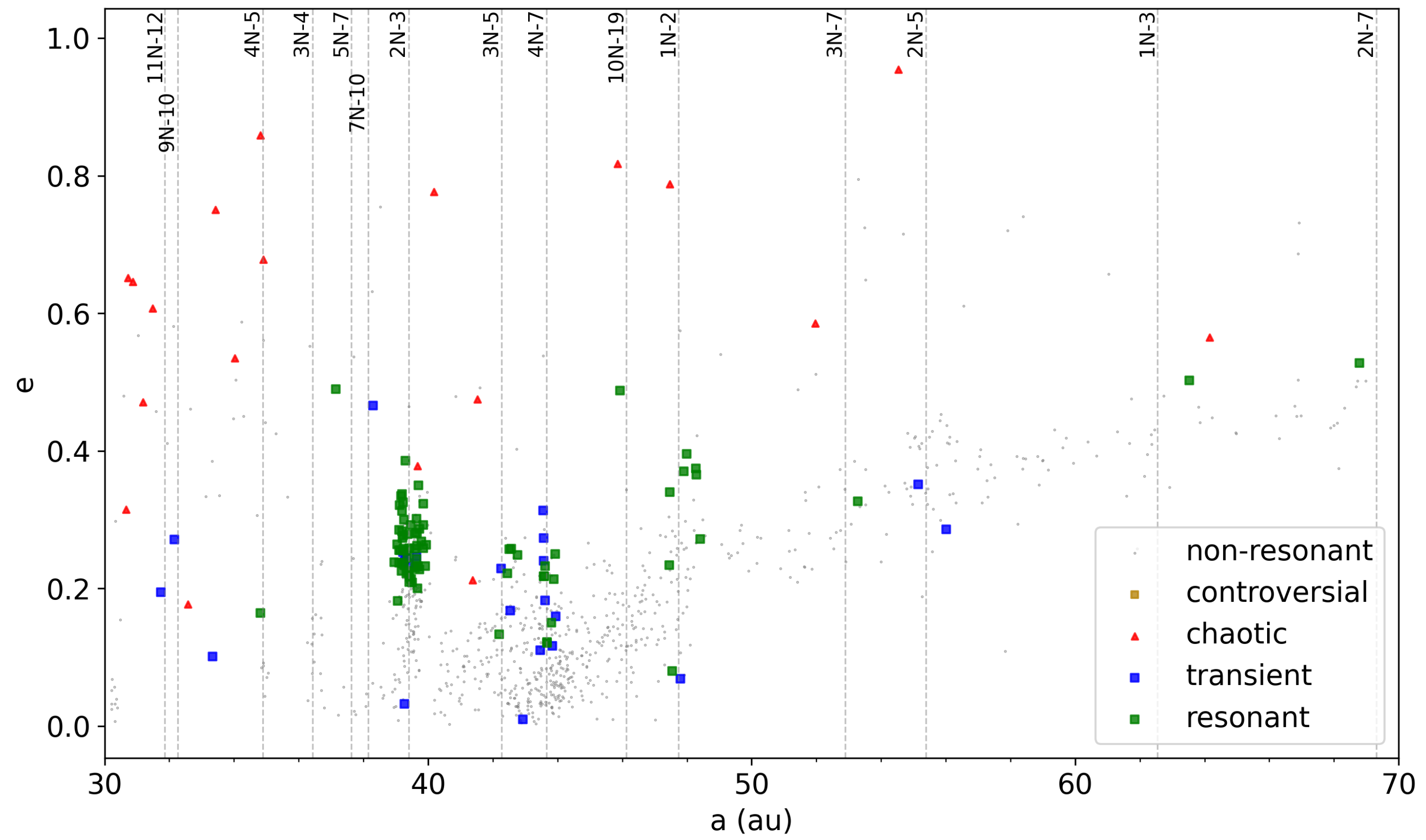
Multiple captures in MMRs of the TNO 523695.

One object, several resonant angles — including **high-order and high-integer MMRs** — with capture, release and re-capture within a single integration.

Smirnov & Žitnik, 2026a, *Icarus*







Dynamically interesting objects.

(652920) 2014 GR₅₃

A distant TNO, $a \approx 202.8$ au — in ZLKR and in **1N-18**.

(523700) 2014 GM₅₄

Low C — **0.54** against 0.92 on average.

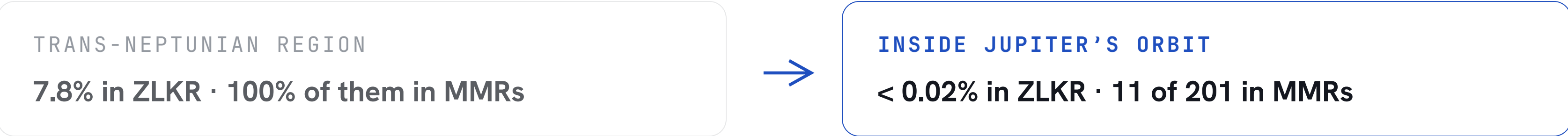
(585899) 2020 HM₉₈

Retrograde, $i \approx 138^\circ$, with a 22 Myr ZLK cycle; trapped in 2N+9, 5N-24, ...

(523695) 2014 GS₅₃

Multiple captures in MMRs, and transient in ZLKR.

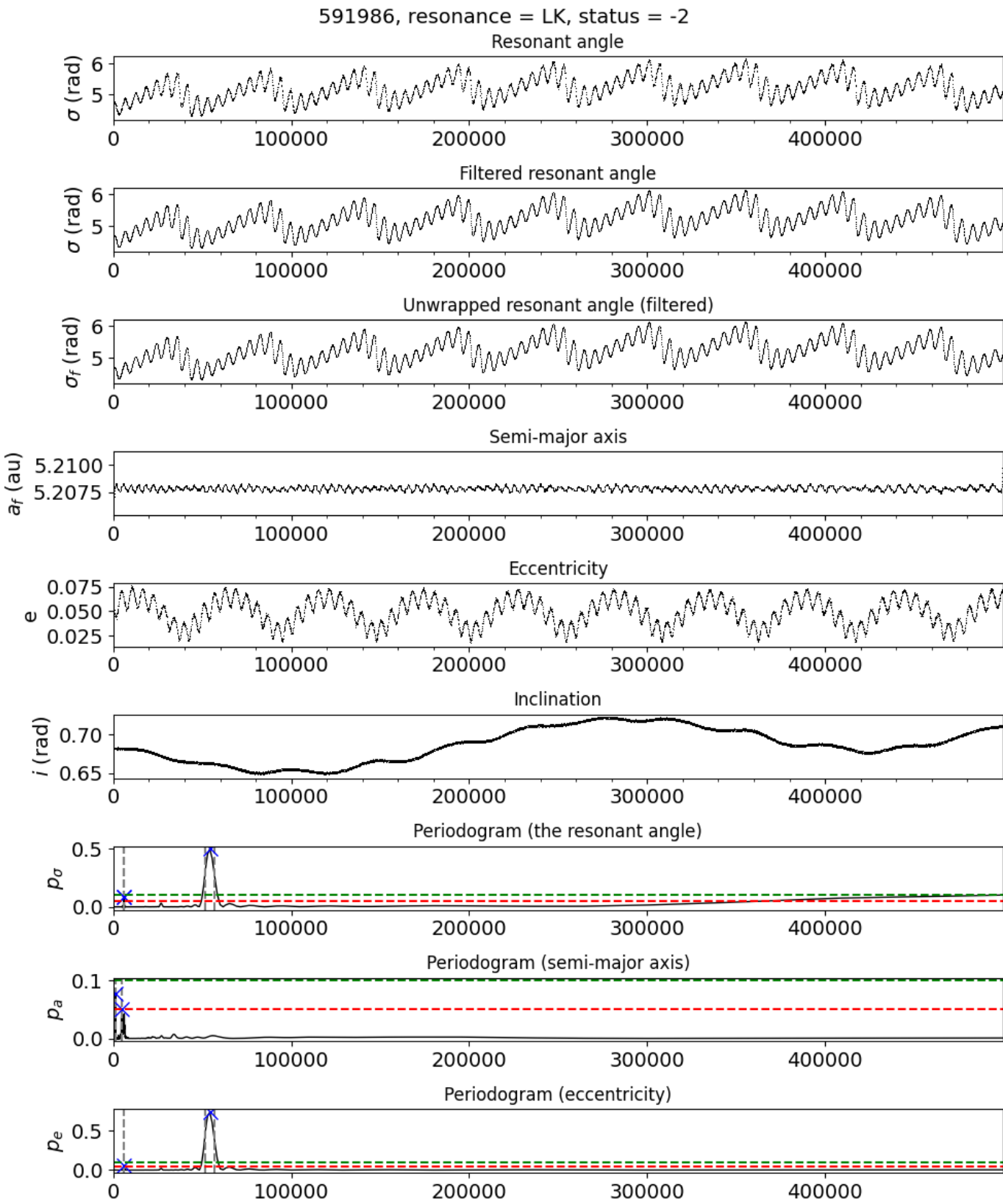
The picture inverts inside 5 au.

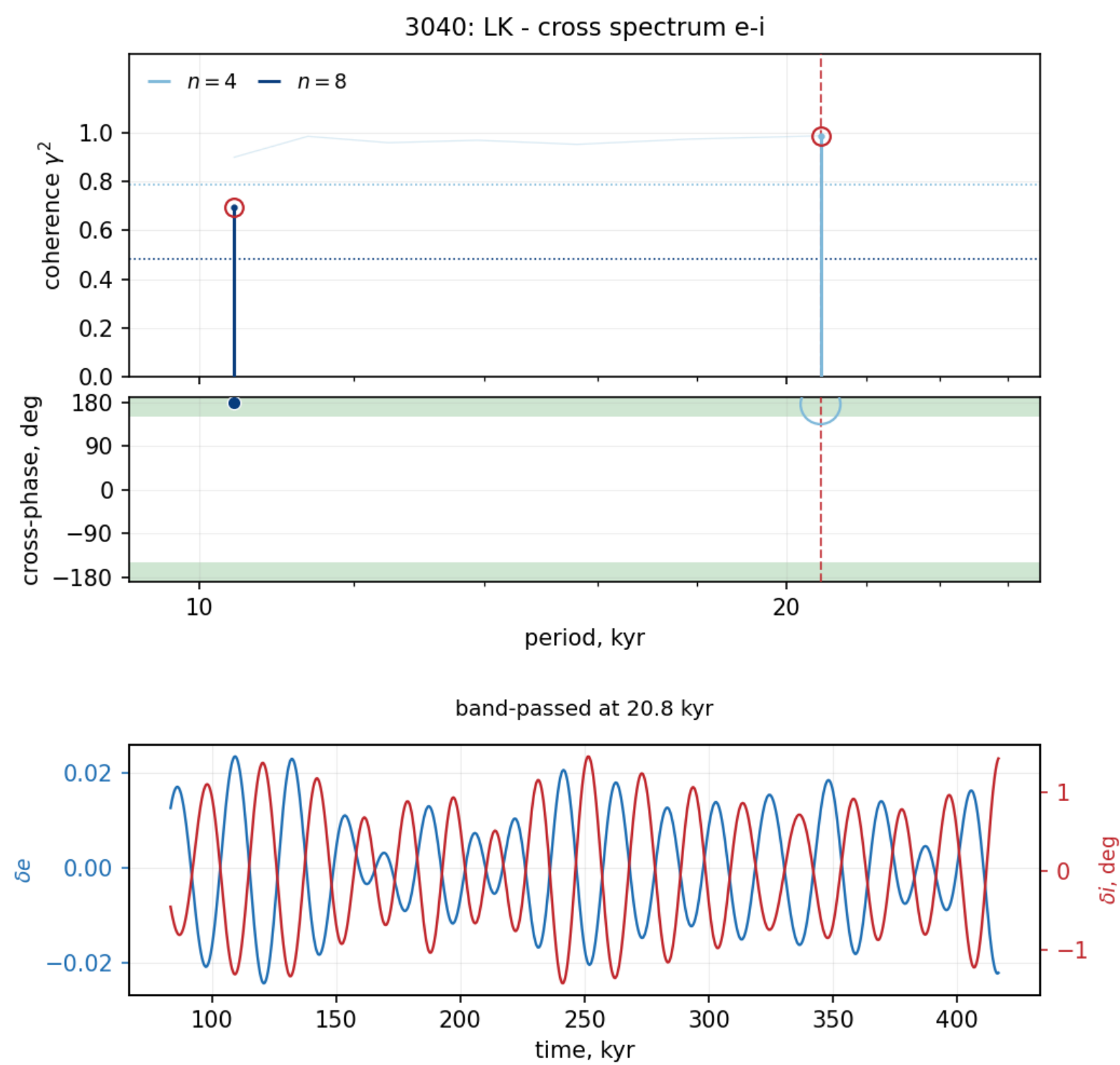


201 of 863 471 asteroids are in ZLKR — < 0.02%	144 near Earth $q < 1.3 \text{ au}$	28 Mars-crossers $1.3 \leq q \leq 1.66 \text{ au}$	11 of the 201 are also in MMRs	26 309 “possibly / temporarily in ZLKR”
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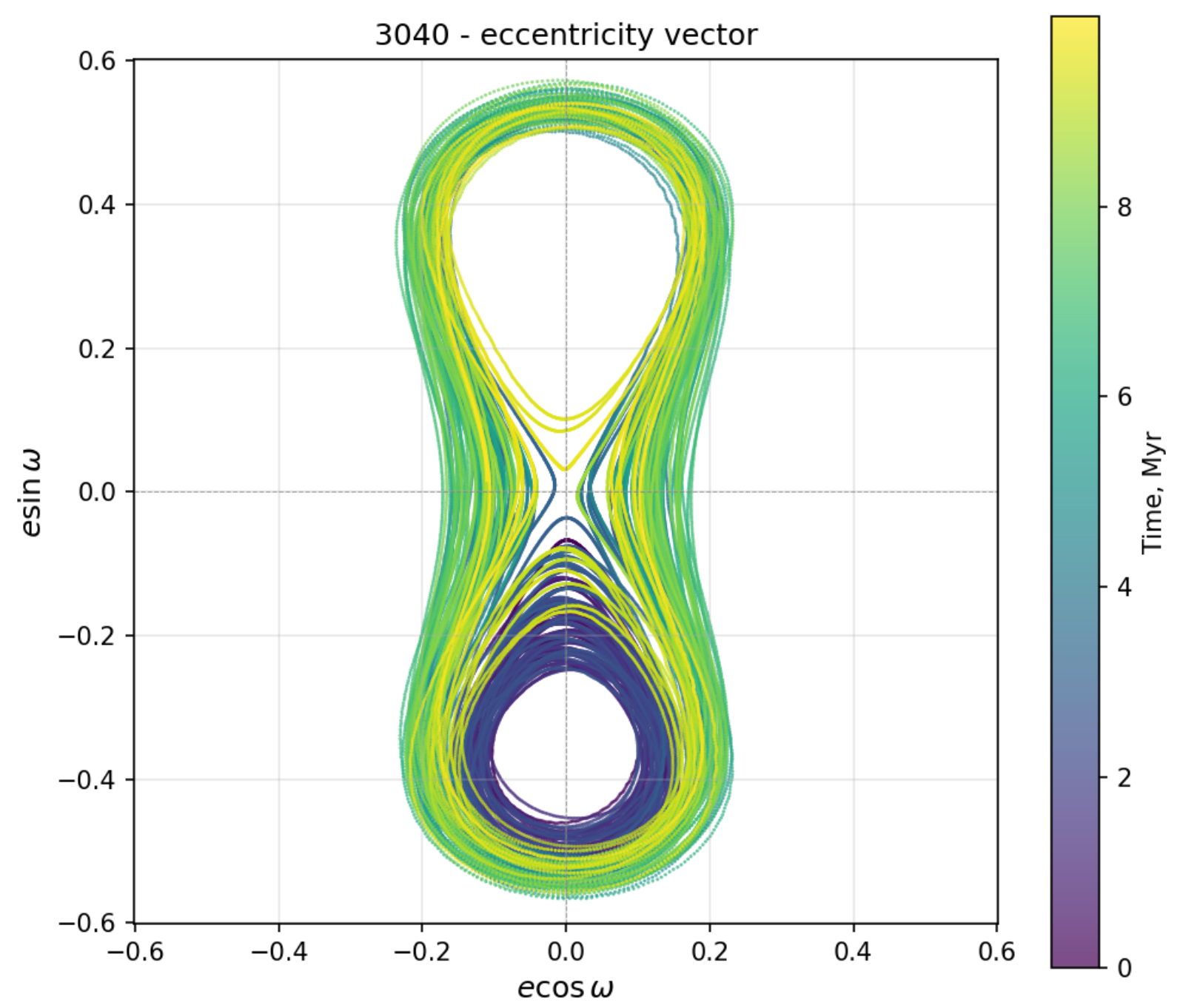
Evolution of the trojan 591986.

Inside Jupiter’s orbit, unlike in the TNO region, there are **false positives** : the resonant angle librates and e and i are anti-correlated — yet physically the object is outside the ZLKR.



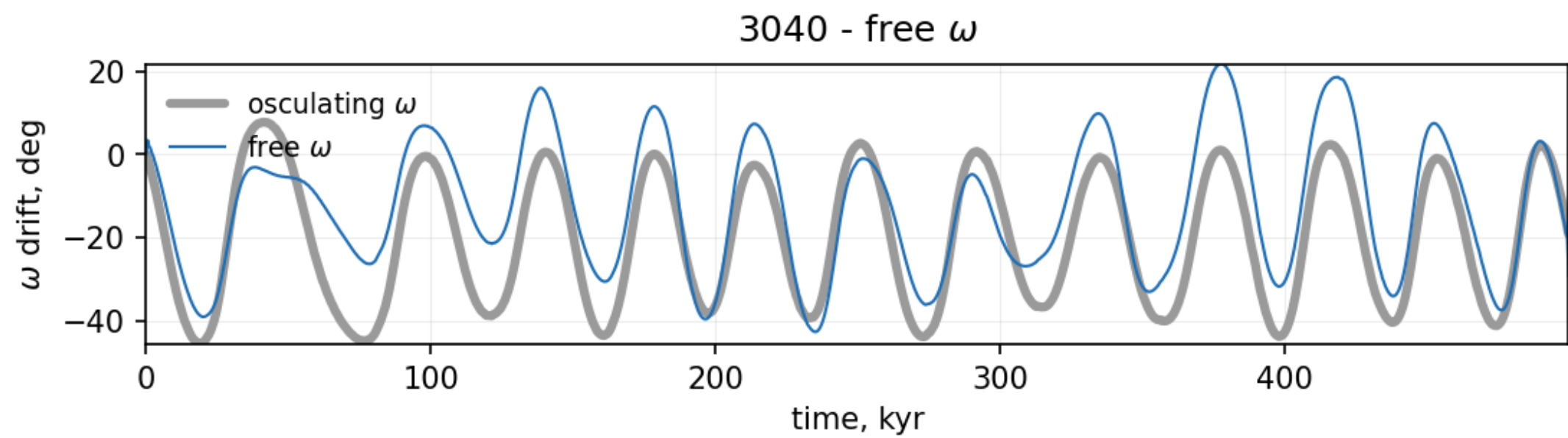


Cross-periodogram of e vs i

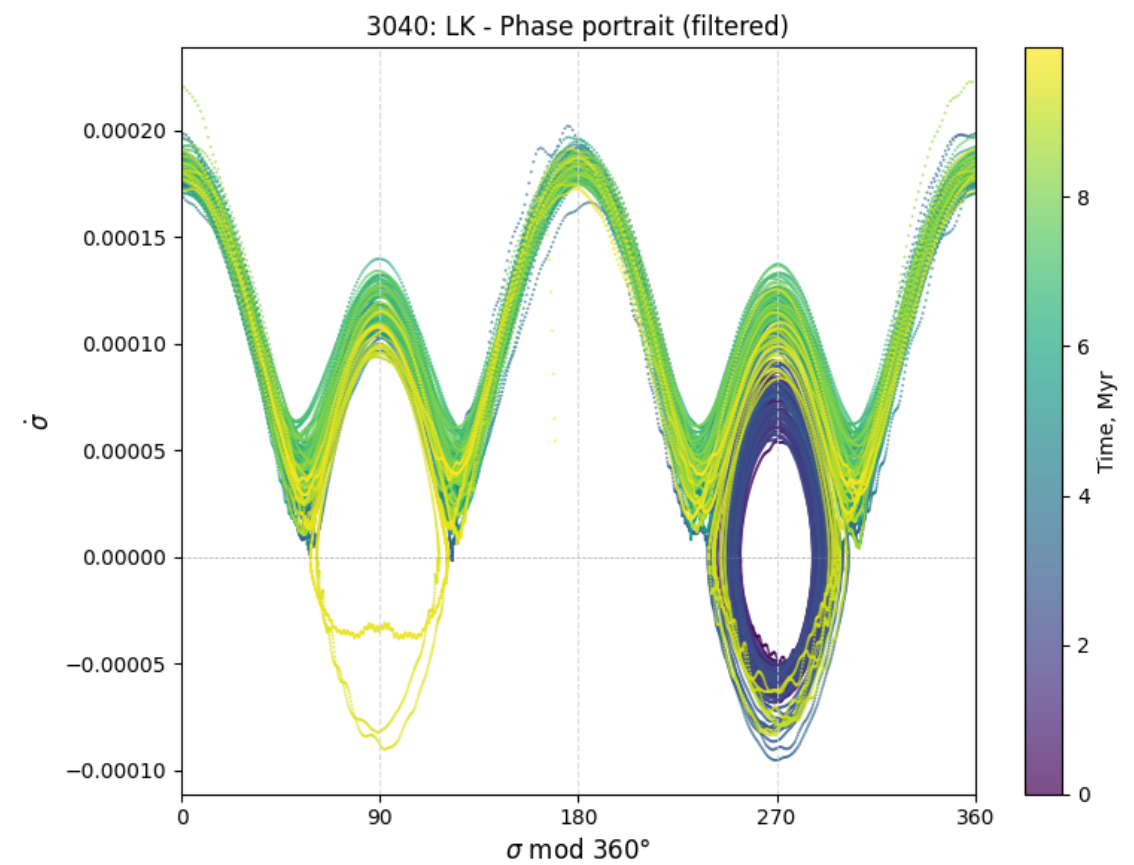
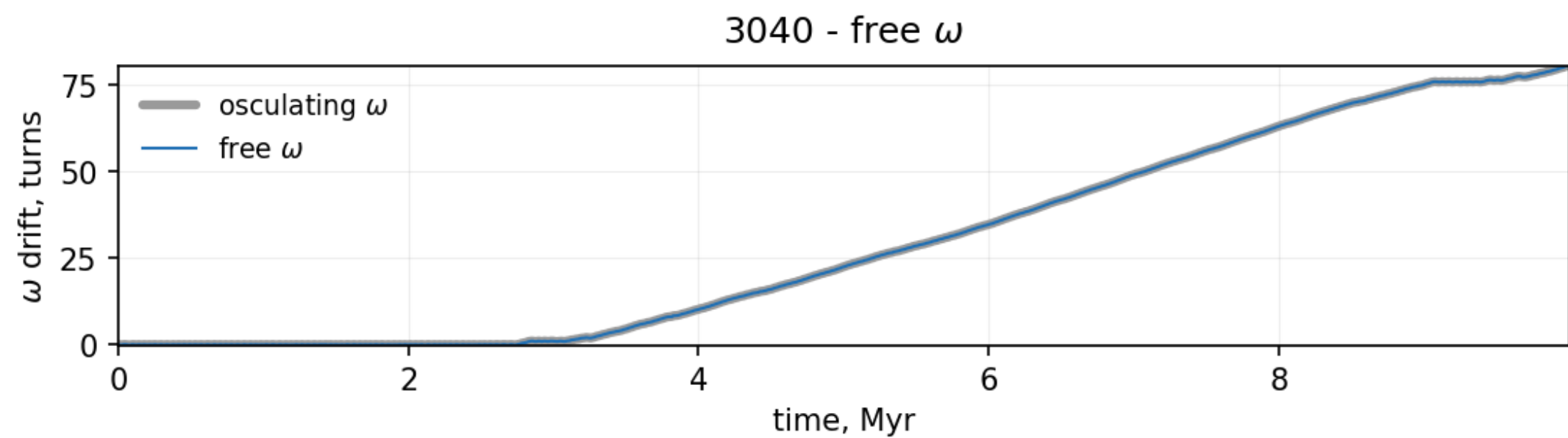


Eccentricity vector

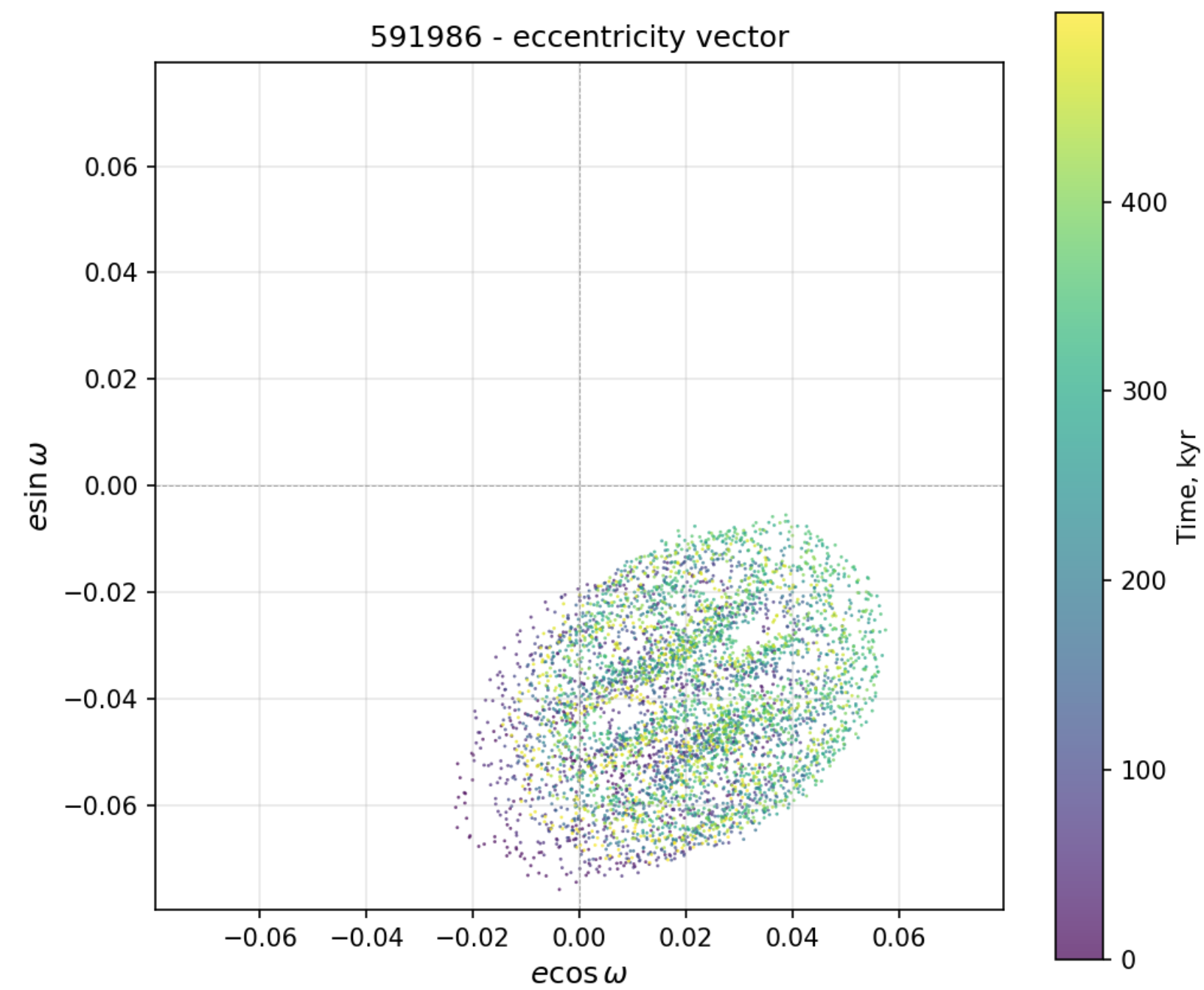
Osculating vs free ω – short window



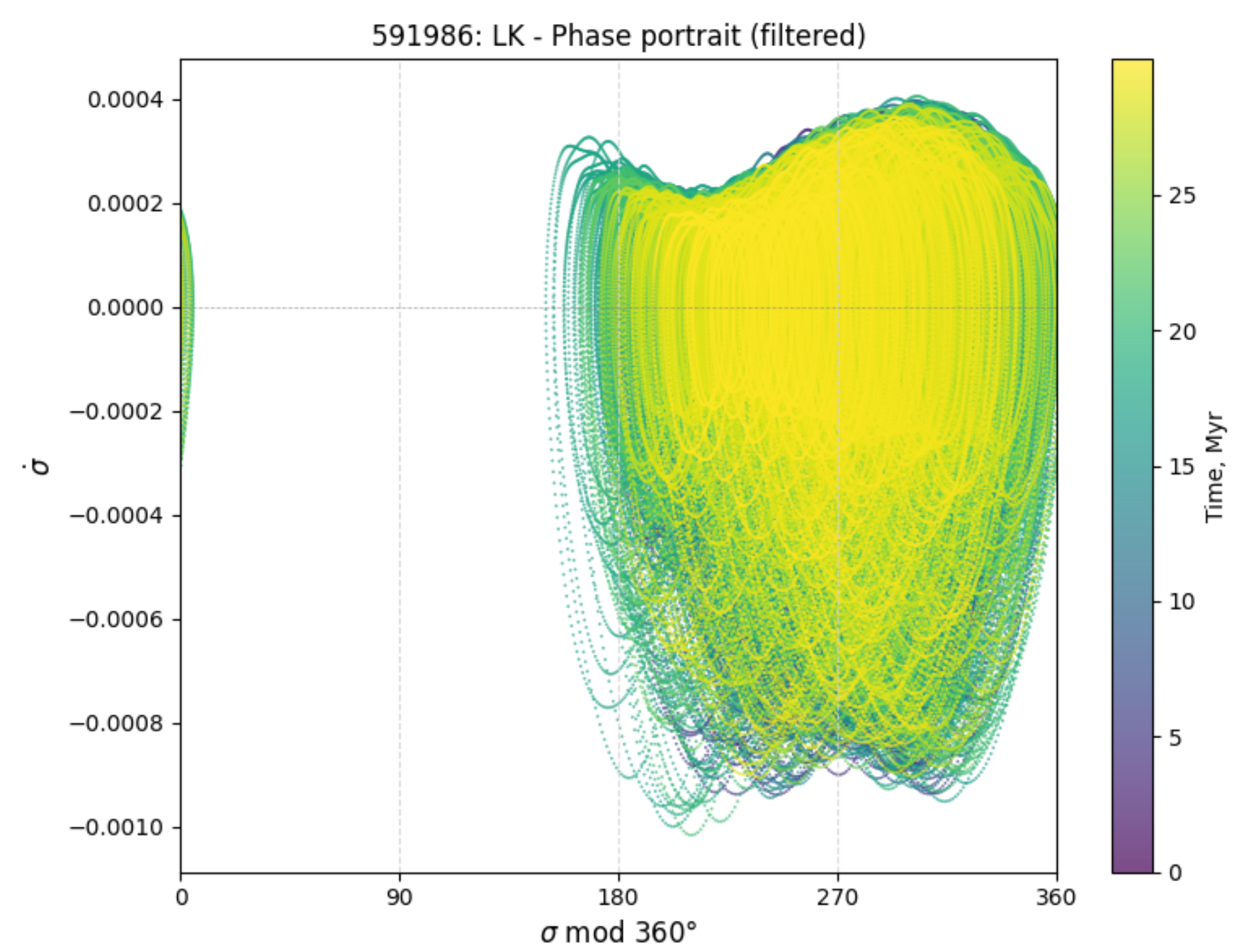
Osculating vs free ω – long window



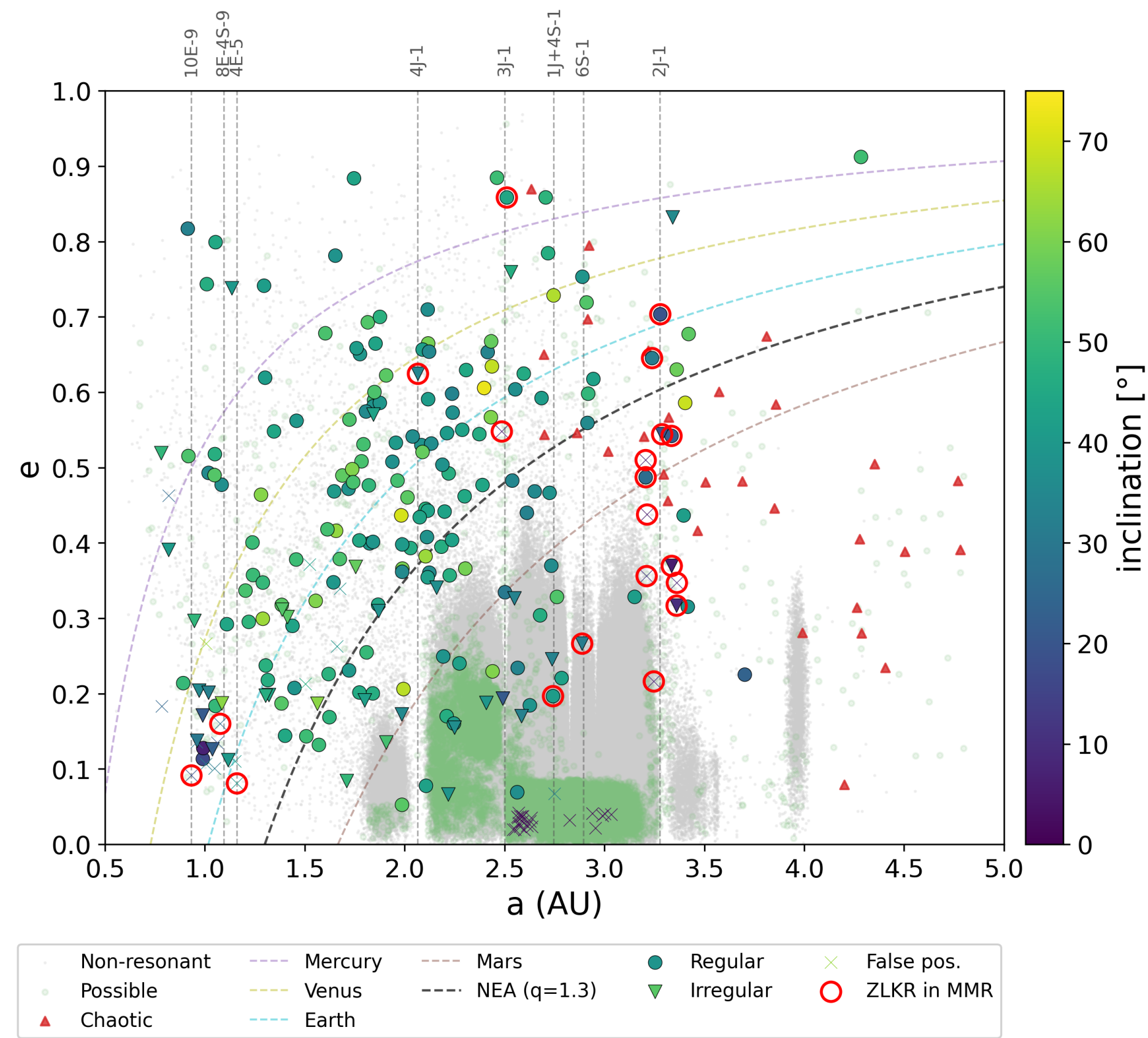
Phase portrait of the filtered σ

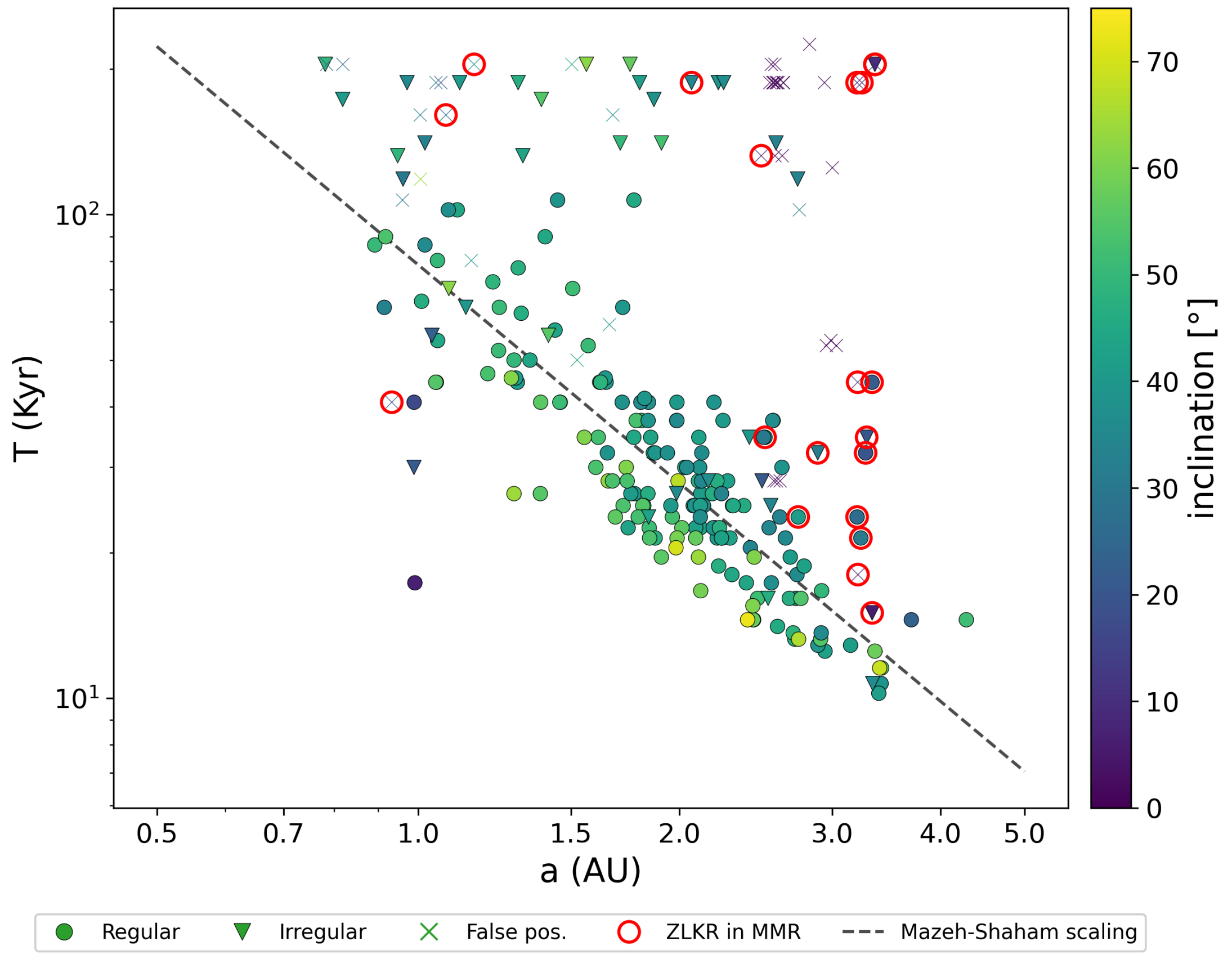


Eccentricity vector



Phase portrait of the filtered σ

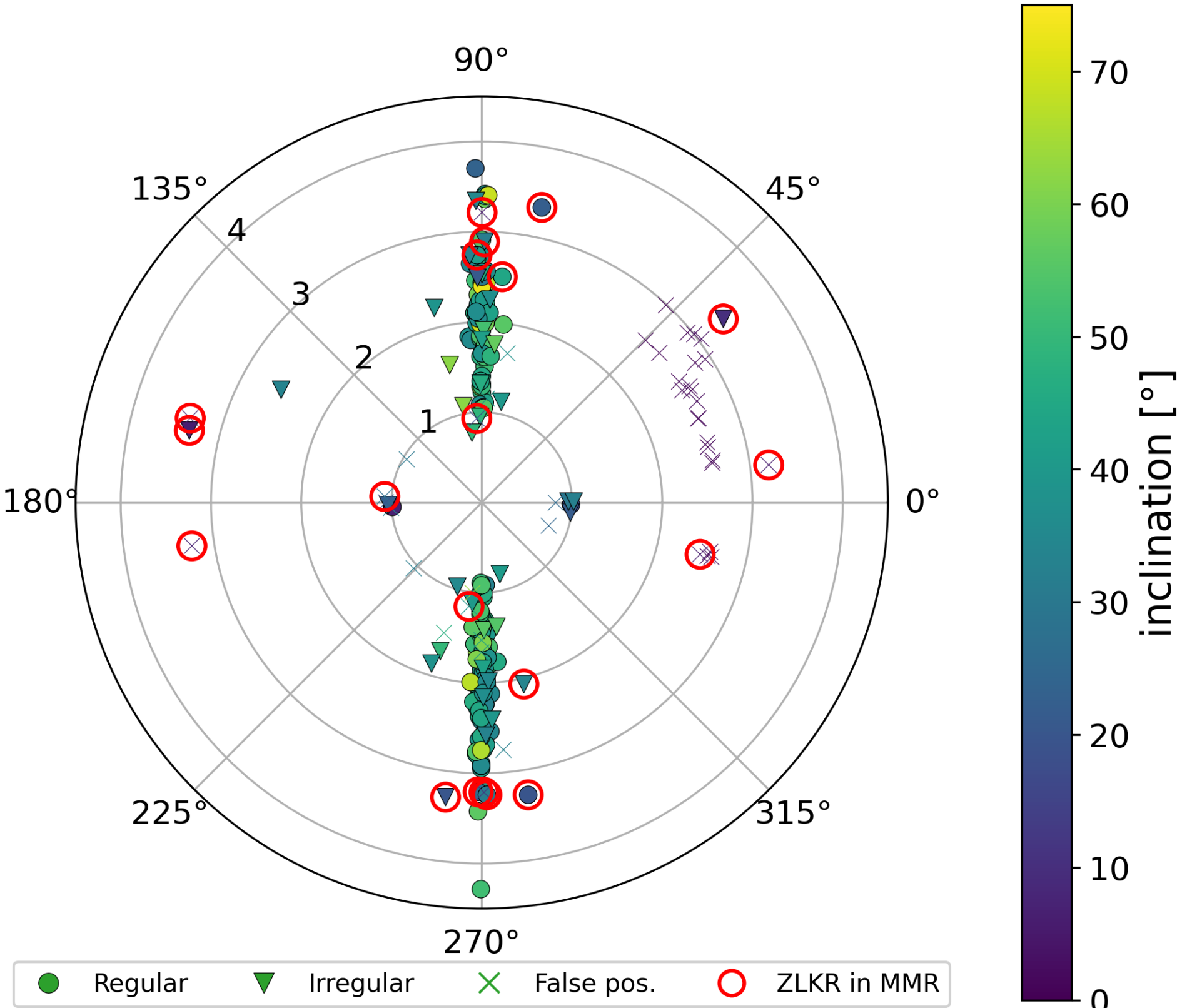




Centers are onto 90° and 270°.

Angle: libration centre. Radius: semi-major axis in au.
Color: inclination.

Smirnov & Žitnik, 2026b, A&A



CONCLUSIONS

- 01 All TNOs trapped in ZLKR are **also trapped in two-body MMRs with Neptune** . For 2N-3, 4N-7, 3N-5 and 4N-5 the libration centres are near 90° and 270° , as expected; for 1N-2 they shift to approximately 120° , 150° , 300° and 330° .
- 02 **1N-X MMRs are indeed strong**: there is a TNO, (652920) 2014 GR₅₃, trapped in 1N-18.
- 03 ZLKRs within 5 au **behave differently** from ZLK capture in the Kuiper belt: an overwhelmingly planet-crossing phenomenon, decoupled from the MMR network, extending below the classical critical inclination.
- 04 The overall number of librators is **significantly lower** than in the TNO region. The interplay and overlapping of different resonances in the main belt appears to suppress long-lived ZLK libration.

01 **The “possible” category and false positives.**
Secular resonances? — and a universal algorithm to decide.

02 **Highly inclined, low-eccentricity objects.**
Outside MMRs and not planet-crossers.

03 **The “interesting” asteroids, one by one.**

04 **Objects with $T_J < 3$.**

THIS IS THE END...

Thank you.

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REFERENCES

Smirnov, E., & Žitnik, I. M. (2026). A systematic survey of von Zeipel-Lidov-Kozai resonances among trans-Neptunian objects: Empirical confirmation of the coupling with mean motion resonances. *Icarus*, 455, 117101. doi.org/10.1016/j.icarus.2026.117101

Smirnov, E., & Milić Žitnik, I. (2026). Identification and dynamical characterization of all numbered asteroids in the von Zeipel-Lidov-Kozai resonance inside Jupiter's orbit. *Astronomy & Astrophysics*. doi.org/10.1051/0004-6361/202660737

Reading list — ZLK resonances in the Solar System readmarginalia.com →