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Artificial Periodic Orbits of a Solar Sail Spacecraft in Moon Vicinity

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SPACECRAFT OPERATING NEAR THE LIBRATION POINTS IN THE EARTH-MOON AND SUN-EARTH SYSTEMS

Up to date, near **20 spacecrafts** operate in the vicinity of the libration points of the Earth-Moon or Sun-Earth system

Spacecraft (origin)	Year	Orbit specification	Mission
ISEE-3 (USA)	1978	L1 Sun-Earth	scientific research
WIND (USA)	1994	L1 Sun-Earth	scientific research
SOHO (USA/EU)	1995	L1 Sun-Earth	scientific research
ARTMIS (USA)	2007	L1/L2 Earth-Moon	probing
HERSHEL (EU)	2009	L2 Sun-Earth	scientific research
Chang'e 2 (PRC)	2010	L2 Sun-Earth	navigation
GAIA (USA)	2013	L2 Sun-Earth	scientific research
Chang'e 5 T1 (PRC)	2014	L2 Earth-Moon	sample return
LISA Pathfinder (EU)	2015	L1 Sun-Earth	trial
DSCOVR (USA)	2015	L1 Sun-Earth	meteorological
Queqiao (PRC)	2018	L2 Earth-Moon	communication satellite



SPACECRAFT OPERATING NEAR THE LIBRATION POINTS IN THE EARTH-MOON AND SUN-EARTH SYSTEMS

Currently, USA, EU, Russia and China proposed plans for **Lunar Space Station** (LSS) creation. The creation of the LSS requires servicing spacecraft, which should best perform the tasks of navigation, communication, telemetry and reconnaissance

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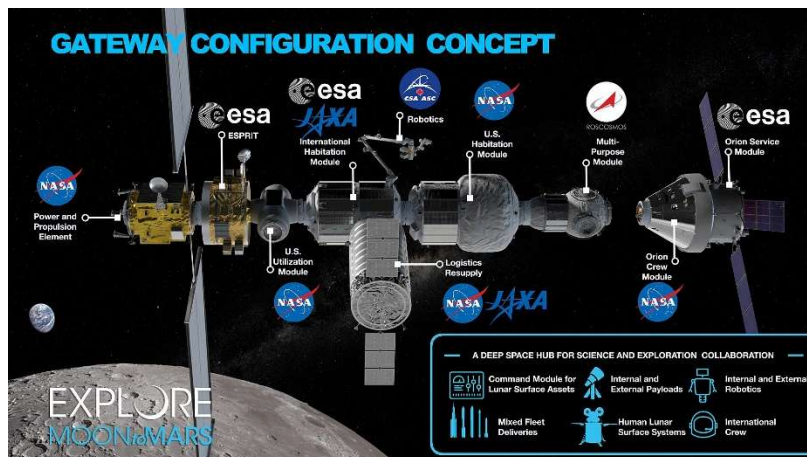


Fig. 1 Future Lunar Space Station and Gateway Station concepts



Relevance of the research topic

1. Lunar exploration programs located on the **far side and south pole of the Moon** need communication and navigation systems
2. **Resonant and cylindrical orbits** in the vicinity of Earth-Moon L2 libration point are convenient to placing such systems, but require fuel consumption to maintaining
3. We can use the solar sail spacecraft (SSSC) to **avoid fuel consumption**
4. The **methodological base is not sufficiently developed** for the formation, maintenance and launch of SSSC into artificial periodic orbits

The existing research:

- Using communication satellites in the Earth-Moon system (S. Gao, J. Duan, L. Zhang);
- Methods for forming orbits of solar sail spacecraft (S. Gong, J. Simo, E.N. Polyakhova);
- Methods for maintaining orbits of solar sail spacecraft (H. Baoyin, A. Farrés, J. Bookless);
- Methods for flight controlling of solar sail spacecraft (O.L. Starinova, J.M. Longuski, M. Macdonald).

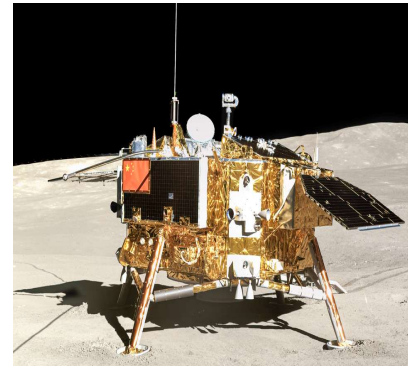


Fig. 2 Chang'e-4 landing on the Moon's far side

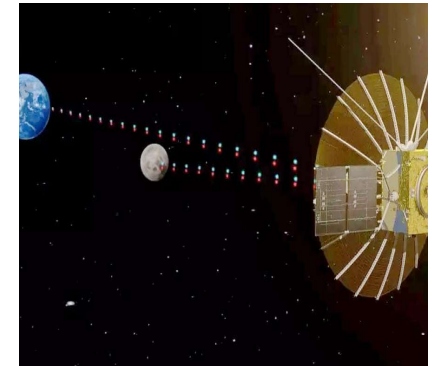


Fig. 3 Queqiao-1 for relay communications on the Moon's far side

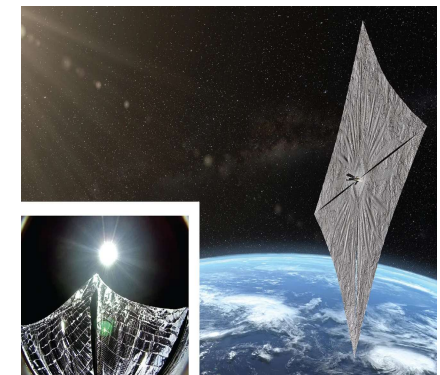


Fig. 4 SSSC using solar light pressure





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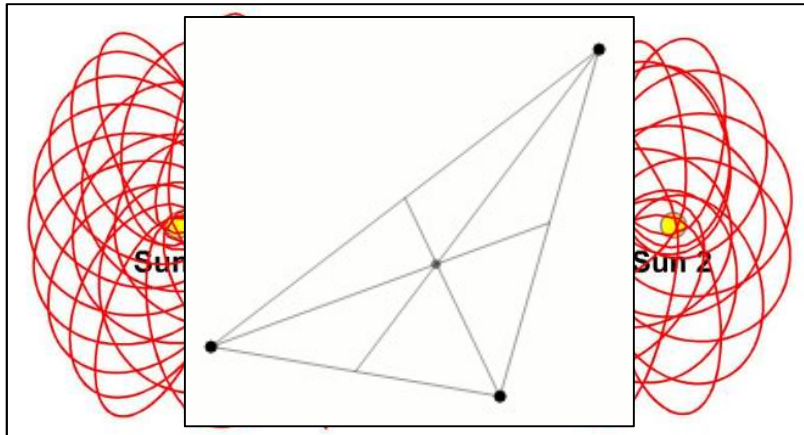
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Simulation results of solar sail spacecraft flights



I. THREE-BODY PROBLEM

In physics and classical mechanics **the three body problem** – is a problem in which the initial positions and velocities of three point masses are known, and their subsequent motions are determined in accordance with the law of universal gravitation. In the general case, it has no solutions in the form of finite analytic expressions.



In theory, three body problem research can be used to study the Earth-Moon or Earth-Sun system movement dynamic.

In engineering, study of the three-body problem is the basis for the construction of ballistic support for missions, which are carried out in the regions of space, where the spacecraft moves under the action of the gravity of several bodies.



I. THREE-BODY PROBLEM

RESTRICTED THREE-BODY PROBLEM AND THE EARTH-MOON SYSTEM

To apply the three-body model to the dynamics of space travel, we use **restricted three body problem**, in which the mass of one body in the system is considered negligibly small relative to the other two bodies. (For example, in the restricted three-body problem consisting of the Earth, the Moon, and the spacecraft, the mass of the spacecraft can be neglected.)

In the restricted three-body problem there exists a special set of solutions in the form of the **libration point**. With the development of space technologies, the motion near the libration point finds more and more practical applications.

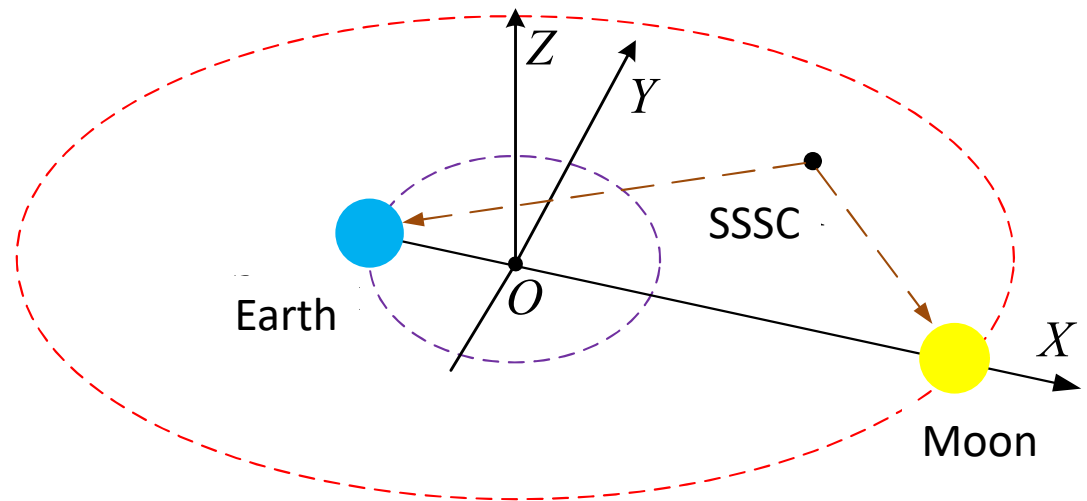


Fig. 5 Earth-Moon system as a restricted three body problem in barycentric rotation frame



I. THREE-BODY PROBLEM

NATURAL AND ARTIFICIAL PERIODIC ORBITS NEAR THE LIBRATION POINTS IN THE EARTH-MOON SYSTEM

Periodic orbits, such as **Lyapunov orbits**, **halo orbits**, **axial orbits**, **vertical orbits**, and others, are potential working orbits of interest for the location of servicing spacecraft.

Periodic orbits whose configuration differs from natural ones are called **artificial periodic orbits**. They can exist without continuous control action. It is precisely such orbits that are considered in this work.

These orbits are precessing with respect to the libration points. The realizations of transfers between such orbits makes it possible to reduce the orbital congestion at the libration points and provide spacecraft movement to different regions of near-lunar space.

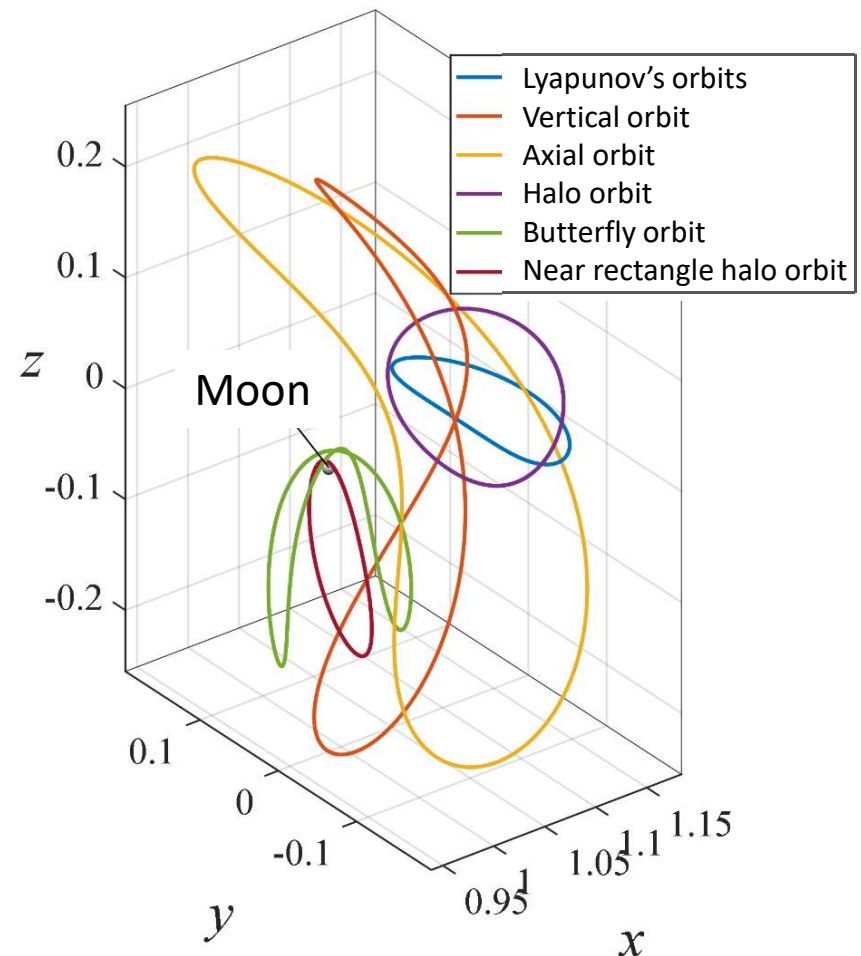


Fig. 6 Natural periodic orbits in the Earth-Moon system



I. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ SPACECRAFT MOTION EQUATION

In the inertial coordinate system, the motion of the spacecraft in near-lunar space is written as follows:

$$\ddot{\boldsymbol{\rho}} = -\frac{\mu_1}{\rho_1^3} \boldsymbol{\rho}_1 - \frac{\mu_2}{\rho_2^3} \boldsymbol{\rho}_2 + \mathbf{f}$$

ρ_1 и ρ_2 - distance between spacecraft and Earth/Moon,

$\mathbf{f}$ - disturbing acceleration,

μ_1 и μ_2 - gravitational parameters of Earth and the Moon

The general equation of motion of the spacecraft in the synodic system:

$$\ddot{\boldsymbol{\rho}} = \dot{\boldsymbol{\Omega}} \times \boldsymbol{\rho} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{\rho}) + 2\boldsymbol{\Omega} \times \mathbf{V} + \mathbf{A}$$

$\dot{\boldsymbol{\Omega}}$ and $\boldsymbol{\Omega}$ angular acceleration and rotation speed of the Moon, relative to the Earth-Moon system barycenter;

$\mathbf{V}$ and $\mathbf{A}$ relative velocity and acceleration of the spacecraft in the Earth-Moon rotational coordinate system.



1. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ SPACECRAFT MOTION EQUATION

For the **circular restricted three-body problem**, the dimensionless equations of motion of the spacecraft in the rotating coordinate system:

$$\ddot{x} - 2\dot{y} = U_x, \quad \ddot{y} + 2\dot{x} = U_y, \quad \ddot{z} = U_z$$

$$U = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} \text{ - system pseudopotential}$$

U_x, U_y, U_z - its partial derivatives

distance : $\rho = l_* r$

velocity : $V = s_* v$

$$\mu = 0,01215 \text{ - Earth and Moon mass ratio}$$

$$\text{time : } T = \frac{\tau_*}{2\pi} t$$

The Earth is located at the point $(-\mu, 0, 0)$

Moon at point $(1-\mu, 0, 0)$

$$r_1^2 = (x + \mu)^2 + y^2 + z^2$$

$$r_2^2 = (x - 1 + \mu)^2 + y^2 + z^2$$

dimensionless distances from the spacecraft to the Earth and the Moon

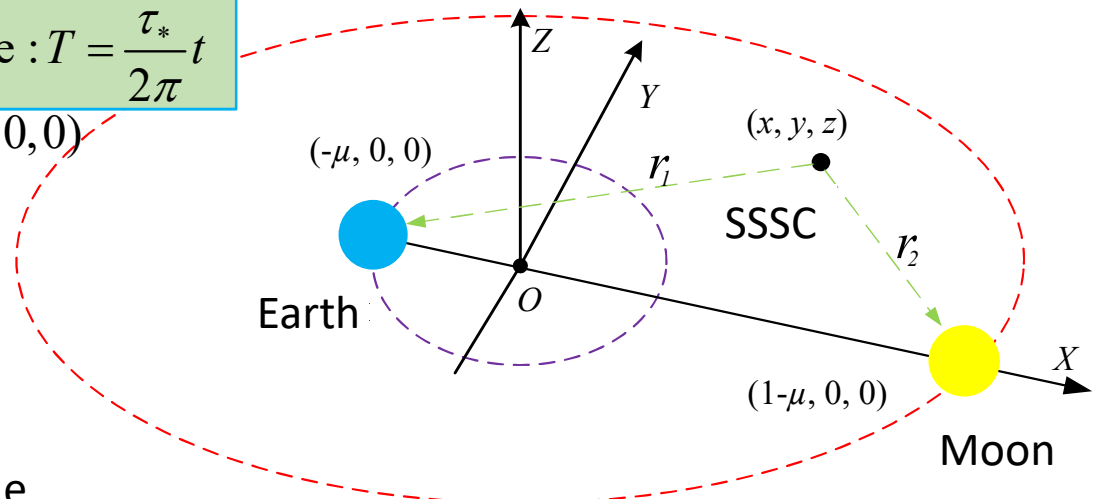


Fig. 7 Synodic dimensionless coordinate system



I. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ LIBRATION POINT

In the **circular restricted three-body problem** there are **libration points**, where spacecraft has zero velocity and zero acceleration, i.e., where the spacecraft will always be at rest relative to celestial bodies (the Earth and the Moon). The points of libration are determined by the conditions:

$$\dot{x} = \dot{y} = \dot{z} = 0$$

$$\ddot{x} = \ddot{y} = \ddot{z} = 0$$

Collinear **L1 L2 L3**

Triangular **L4 L5**

Revolving around the Earth with the Moon

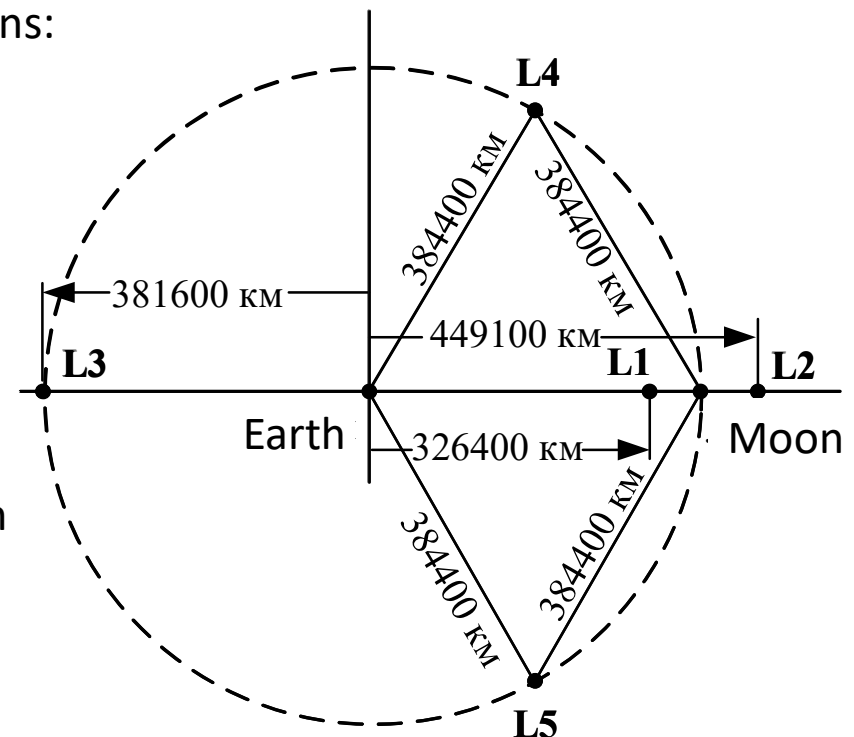


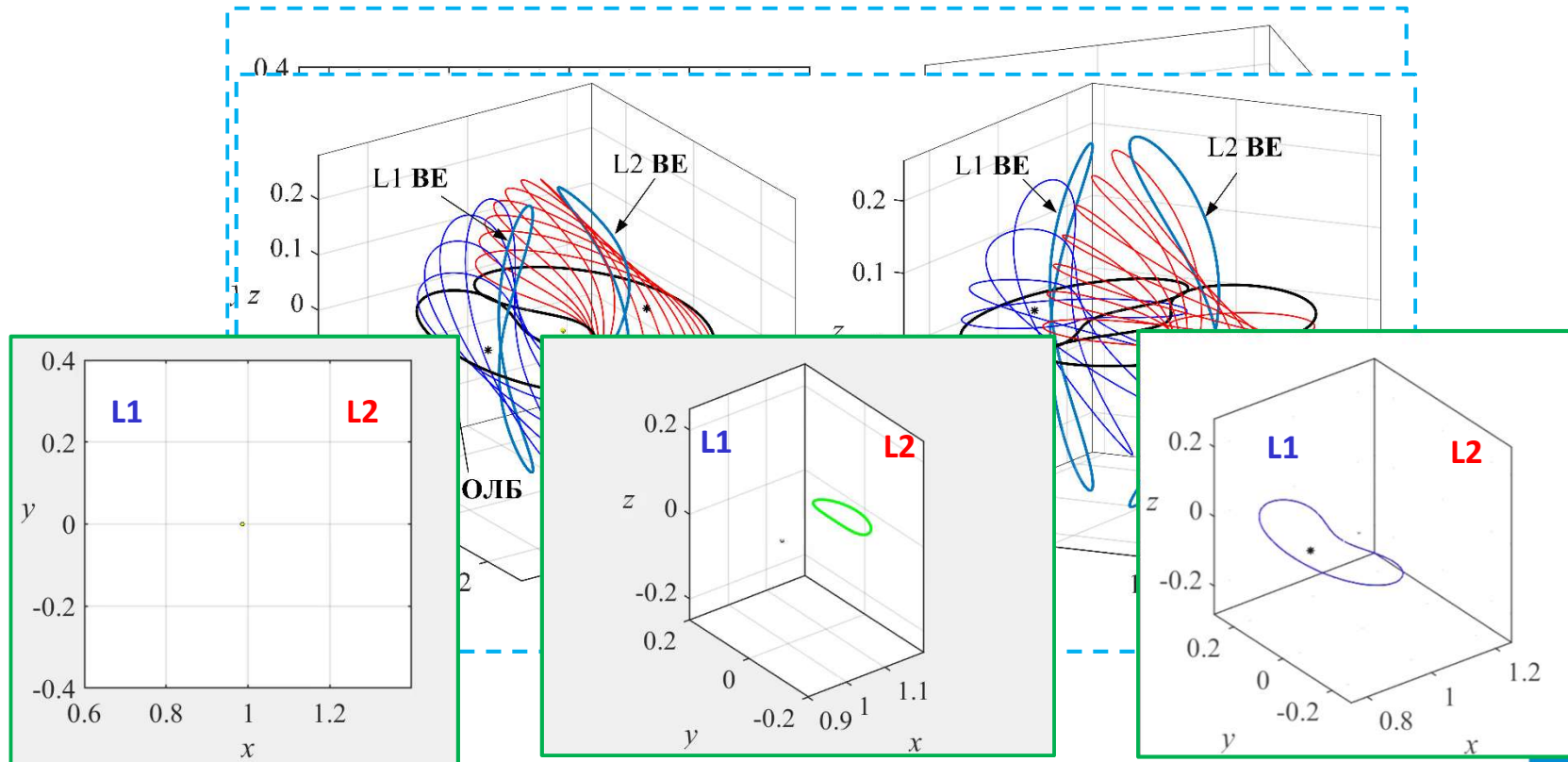
Fig. 8 Distributions of libration points



1. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ PERIODICAL ORBITS

The main focus of this work is on the **Lyapunov orbits (OL)**, **halo orbits (HA)**, **axial orbits (OS)** and **vertical orbits (VE)** relative to the points **L1** and **L2** of the Earth-Moon system.





I. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ PERIODICAL ORBITS parametrization

Since these orbits are calculated **numerically**, in this paper periodic orbits are evaluated in terms of the following **parameters**:

Lyapunov orbit

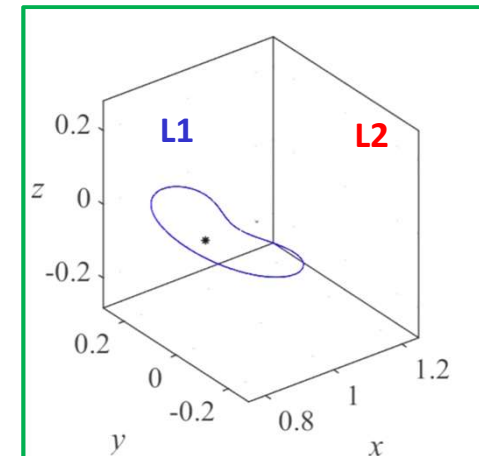
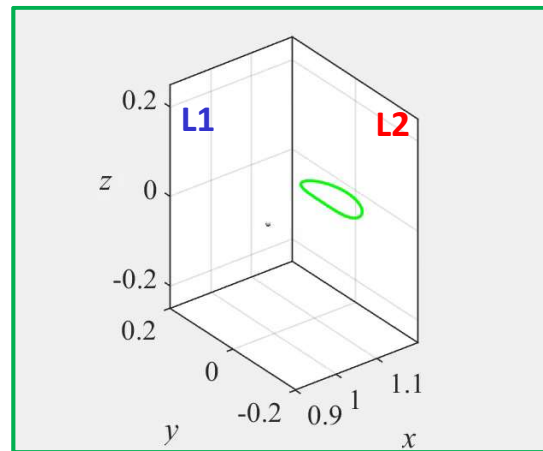
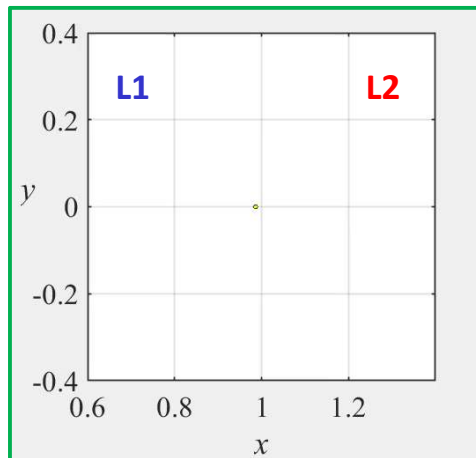
minimum distance $\min|x|$ from the orbit to the Moon

Halo orbit

maximum distance $\max|x|$
out-of-plane amplitude $|Az|$

Axis orbit

out-of-plane amplitude $|Az|$





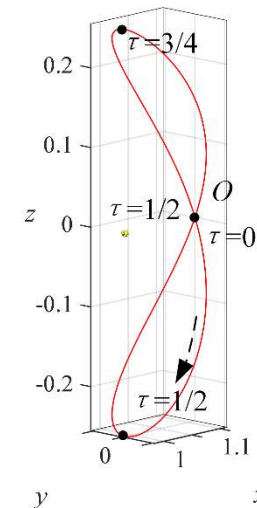
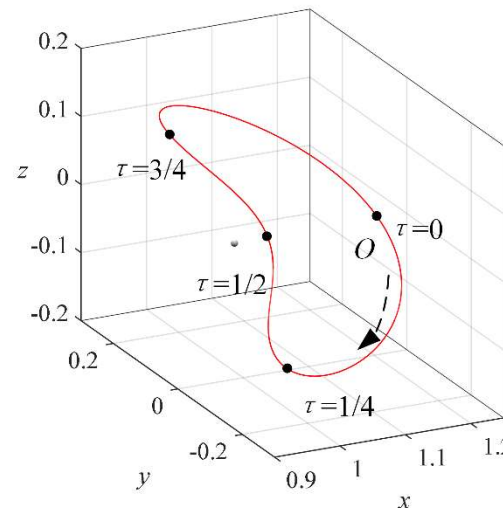
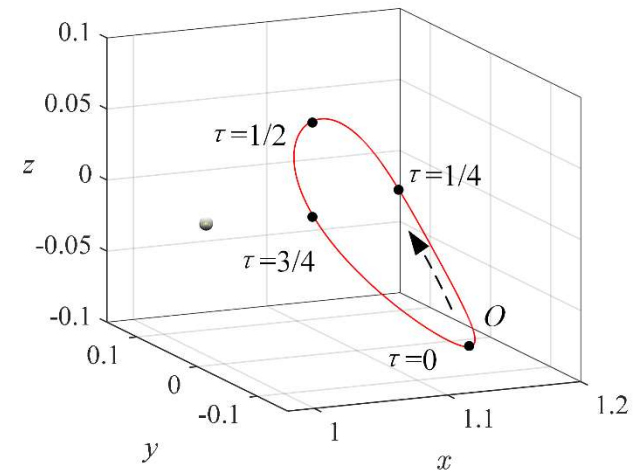
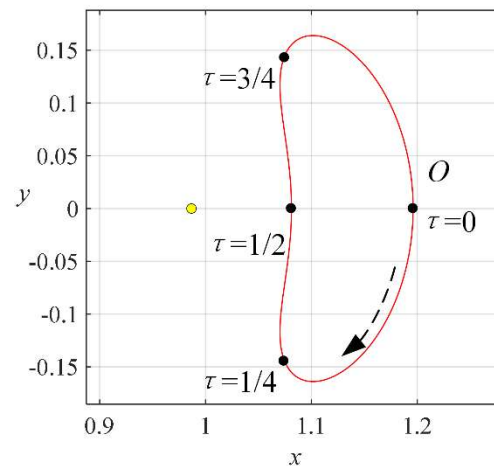
1. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ PERIODICAL ORBITS parametrization

To specify the position of the spacecraft on periodic orbits for subsequent calculations of the flight, we assume:

$$\tau = t / T_0$$

- the ratio of the spacecraft flight time from point O (the farthest point in the orbit from the Moon) to the current orbital position divided by orbital period





I. THREE-BODY PROBLEM SPACECRAFT MOTION IN THE EARTH-MOON SYSTEM

◆ JACOBI CONSTANT

In celestial mechanics, **Jacobi constant** (Jacobi integral) is the only known conserved variable within the circular restricted three-body problem. It can be applied as an **energy** of the moving spacecraft in a three-body system

$$J = 2U - v^2$$

where $v^2 = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$

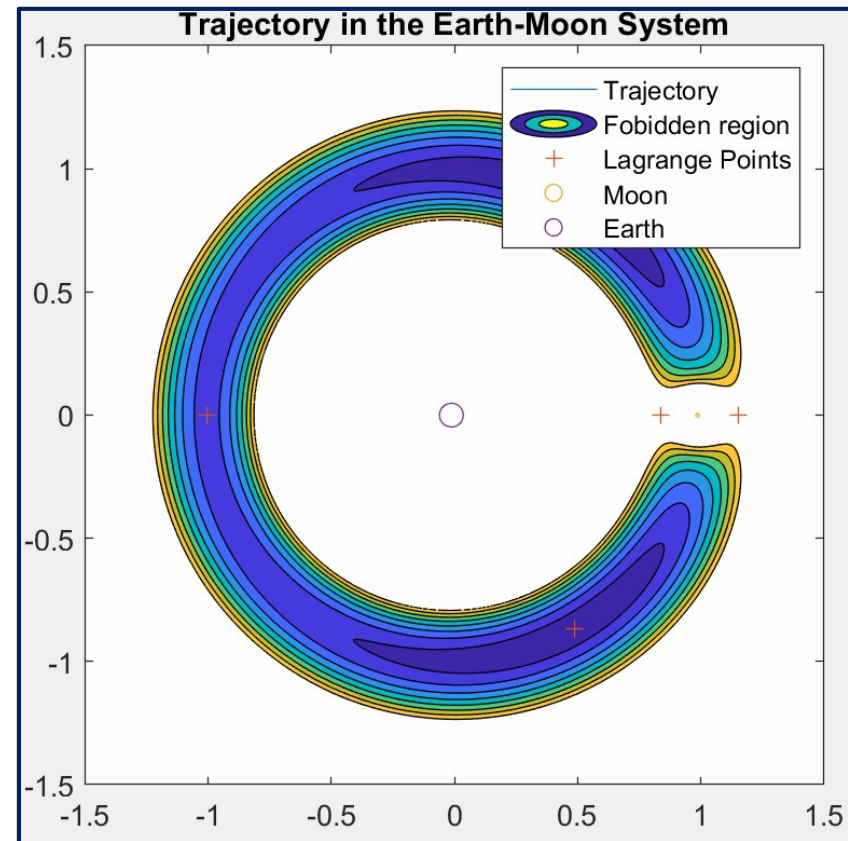


Fig. 9 Jacobi integral in synodic dimensionless coordinate system



2. Mathematical models of SSSC motion. Acceleration from light pressure

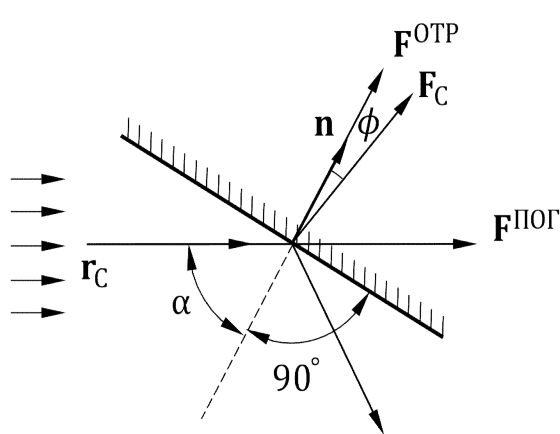


Fig. 10. Scheme of the sunlight effect on a SSSC

$\mathbf{F}^{\text{OTP}}$ – produced by reflected light $\mathbf{F}^{\text{PIOT}}$ – produced by absorbed light

$\mathbf{r}_c$ – sunlight vector direction $\mathbf{n}$ – vector of the SS normal

ϕ – angle between the SS normal and the light pressure

Acceleration of the SSSC with controlled reflective ability :

$$\mathbf{a}_c = \frac{u\mathbf{F}^{\text{PIOT}} + (1-u)\mathbf{F}^{\text{OTP}}}{m} = \kappa \left[\frac{u}{2}(\mathbf{r}_c \mathbf{n})\mathbf{r}_c + (1-u)(\mathbf{r}_c \mathbf{n})^2 \mathbf{n} \right] \quad (1)$$

$\kappa = 2 \frac{\varepsilon S}{cm}$ – the value of the SSSC nominal acceleration;

ε – sunlight energy density;

c – the light speed;

S – surface area of SS;

m – spacecraft mass.

Table 1 Values of nominal acceleration of typical SSSC

SS projects	Ikaros	LightSail-2	Solar Cruiser	Sunjammer	SS in this work
Mass of the SSSC, kg	310	5	90	32	200
SS area, m^2	196	32	1672	1200	3582
Value κ , mm/s^2	0.005	0.058	0.169	0.340	0.163



2. Mathematical models. Acceleration from light pressure

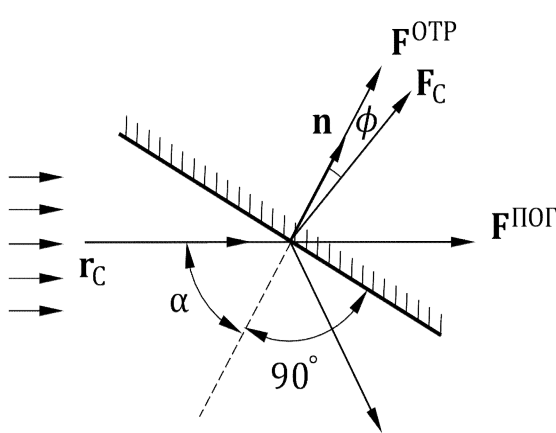


Fig. 11. Scheme of the sunlight effect on a SSSC

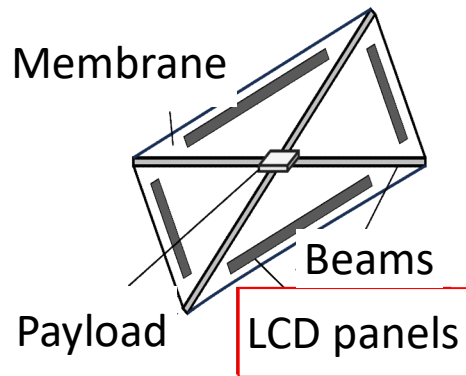


Fig. 12. Design of the SS with controlled reflective ability

$\mathbf{F}^{\text{OTP}}$ – produced by reflected light $\mathbf{F}^{\text{IIOT}}$ – produced by absorbed light

$\mathbf{r}_c$ – sunlight vector direction $\mathbf{n}$ – vector of the SS normal

ϕ – angle between the SS normal and the light pressure force

Acceleration of the SSSC with controlled reflective ability

$$\mathbf{a}_c = \frac{u\mathbf{F}^{\text{IIOT}} + (1-u)\mathbf{F}^{\text{OTP}}}{m} = \kappa \left[\frac{u}{2}(\mathbf{r}_c \mathbf{n})\mathbf{r}_c + (1-u)(\mathbf{r}_c \mathbf{n})^2 \mathbf{n} \right] \quad (1)$$

$u \in [0, 1]$ – reflectivity, which varies depending on the area of the turning on LCD panels.

If the SS installation angle $\alpha \neq 0$ and the reflection coefficient $u \neq 0$, the sunlight direction does not coincide with the direction of the SS normal (**Fig. 13**).

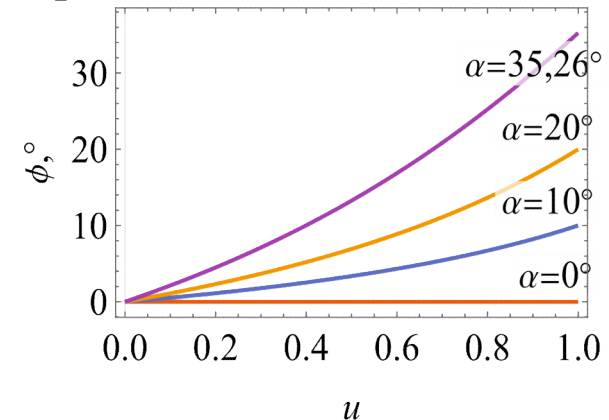


Fig. 13 Change of the angle ϕ from the reflection coefficient u and the installation angle α



2. Mathematical models. Equations of spacecraft motion

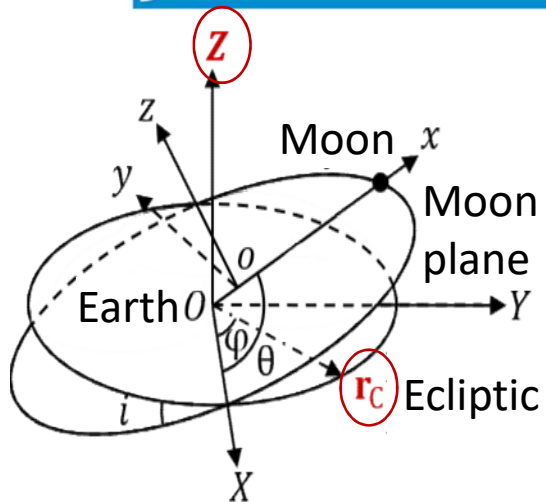


Fig. 14 Coordinate systems (CS) taking into account Sun, Earth and Moon motions

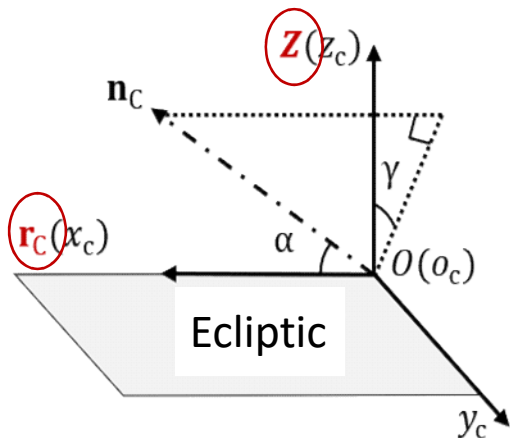


Fig. 15 Coordinate system for describing the SS position

$OXYZ$ – geocentric inertial frame; $oxyz$ – barycentric rotating frame;

$\varphi(t) = \varphi_0 + \omega_3 t$ – angle of sunlight direction in the system $OXYZ$;

ω_3 – Earth's angular velocity;

$\theta(t) = \theta_0 + \omega_{\text{J}} t$ – angle of the Moon's position;

ω_{J} – Moon's angular velocity;

$i = 5.145^\circ$ – overage inclination of the Moon's orbital flat.

1. Equation of SC motion in the $oxyz$ coordinate system:

$$\ddot{\mathbf{r}} + 2\boldsymbol{\omega}_{\text{J}} \times \dot{\mathbf{r}} = \nabla U(\mathbf{r}) + \mathbf{a}_{\text{CB}} \quad (3)$$

$$\mathbf{r} - \text{SC position vector; } U(\mathbf{r}) = \frac{1}{2}(\boldsymbol{\omega}_{\text{J}} \times \mathbf{r})^2 + \frac{1-\mu}{r_1} + \frac{\mu}{r_2}$$

Dimensional quantities:

$$[L] = l_{3\text{J}} = 3,84 \times 10^8 \text{ m} \quad [M] = m_3 + m_{\text{J}} = 6,01 \times 10^{24} \text{ kg} \quad [T] = 1/\omega_{\text{J}} \approx 4,35 \text{ days.}$$

2. The direction vector of the SS normal (Fig. 15):

$$\mathbf{n}_c = [\cos\alpha \quad \sin\alpha \sin\gamma \quad \sin\alpha \cos\gamma]^T \quad (4)$$

$O_c x_c y_c z_c$ – coordinate system relative to sunlight



3. Statement of the optimal control problem. Forming of the second order resonant orbits

The **second order resonant orbits** are a orbital type, for which the flight duration along two turns equals to the period of the Sun relative motion around the Earth-Moon system T_c .

The **task of forming resonant orbits** is to determine the initial conditions that ensure the closure of the spacecraft's flight path over a period using natural orbits whose period is $T_c/2$ as initial approximations for a given sail and control parameters.

Assumptions at the orbit forming stage :

1. SS has an ideal reflective surface ($\mathbf{u}=\mathbf{0}$);
2. The installation angles α and γ are fixed;
3. The Moon's orbital inclination to the ecliptic is not taken into account $\mathbf{i}=\mathbf{0}$, which allows us to consider the sunlight direction changing as periodic:

$$\mathbf{r}_{CB} = [\cos\lambda \quad \sin\lambda \quad 0]^T \quad (4)$$

$\lambda(t) = \lambda_0 - \omega_c t$ – the sunlight direction angle;

ω_c – the relative Sun's angular velocity

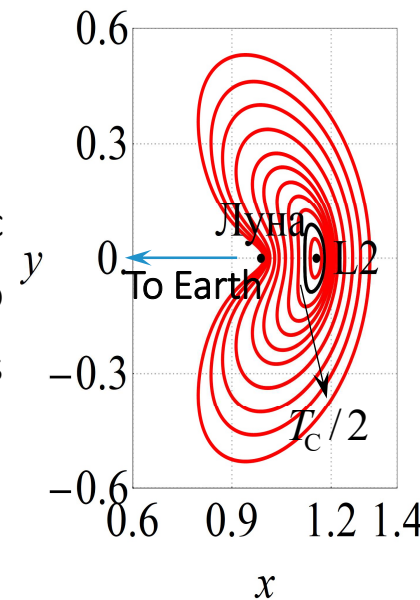


Fig. 16 Natural periodic
Lyapunov's orbits

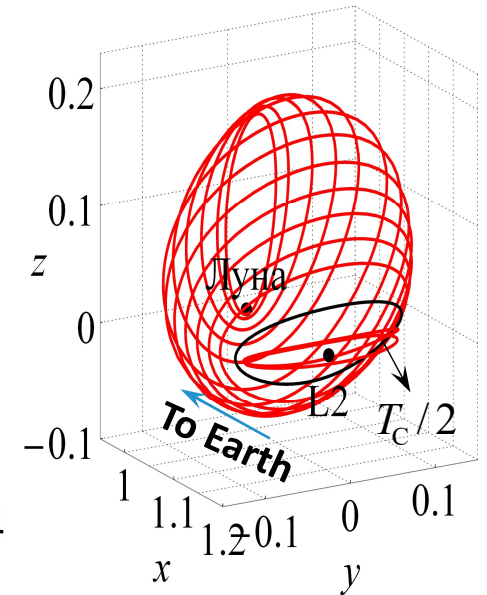


Fig. 17 Natural
periodic halo orbits



3. Forming of the second order resonant orbits

Method and calculation technic for resonant orbits formation (**Fig. 18**):

1. The multiple firing method is used to determine the initial conditions that ensure the closure of the spacecraft's flight path over a period of $[0, T_C]$:

1.1. The interval is divided into N parts:

$$0=t_0 < t_1 < t_2 < \dots < t_N = T_C$$

1.2. The state vector is adjusted so that

$$\mathbf{X}(t_0) = \mathbf{X}(t_N)$$

2. The parameter continuation method is used to transform a natural orbit with a period of $T_C/2$ into a SSSC orbit under the influence of solar pressure: $\kappa^j = \kappa^{j-1} + \Delta\kappa$

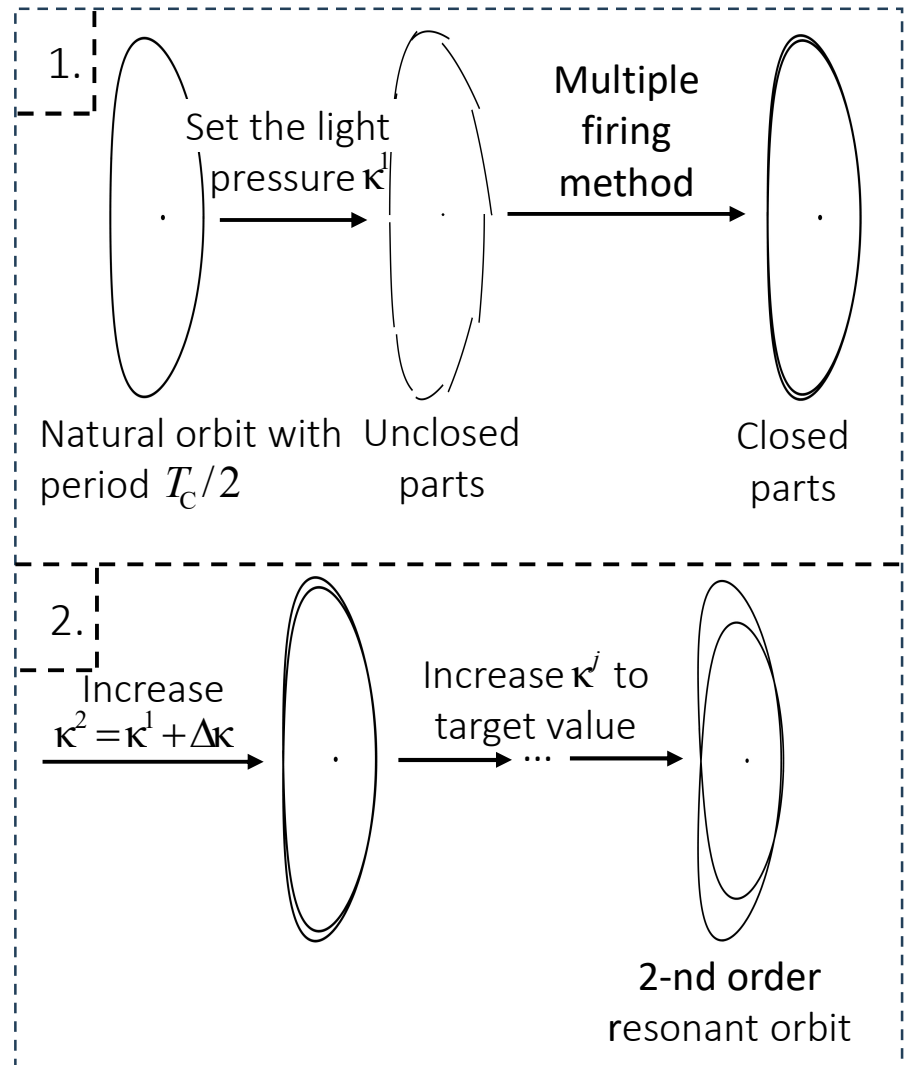


Fig. 18 Scheme of operation of the resonant orbit formation technique



3. Forming of the second order resonant orbits

Analysis of the parameters influence on the resonant orbits geometry :

1. the angle of the initial Sun position λ_0 ;
2. the nominal acceleration value κ ;
3. the **sail installation** angle α .

Resonant orbits derived from Lyapunov's orbits are called **Lyapunov conformal**;

Resonant orbits derived from halo orbits are called **halo conformal** orbits.

1. The orbital geometry for difference angle of the initial Sun position λ_0

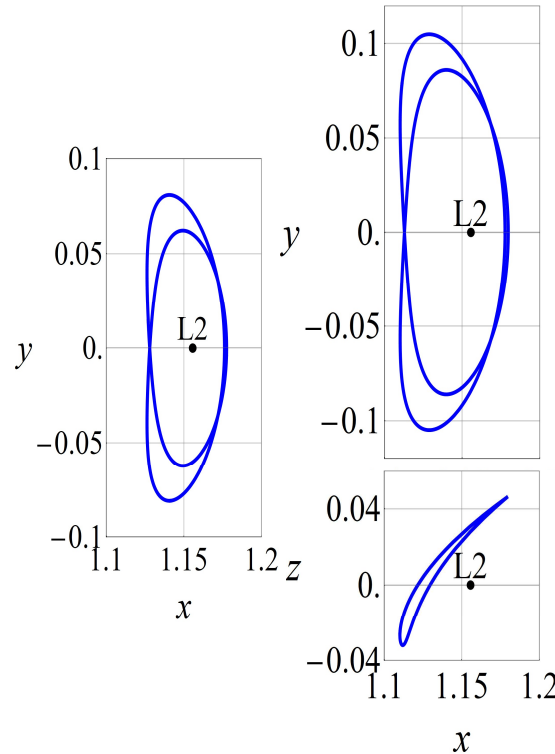


Fig. 19 Lyapunov conformal (left) and **halo conformal** (right) orbits first configuration
 $\lambda_0 = 0^\circ$ or 180°

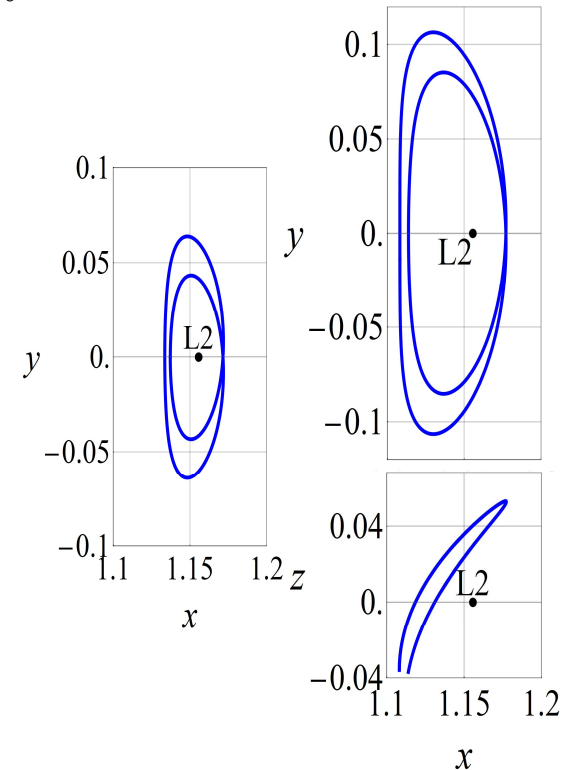


Fig. 20 Lyapunov conformal (left) and **halo conformal** (right) orbits second configuration
 $\lambda_0 = 90^\circ$ or 270°



3. Forming of the second order resonant orbits

2. Influence of the nominal acceleration value κ

$\kappa = 0$ $\kappa = 0.008$ $\kappa = 0.0016$

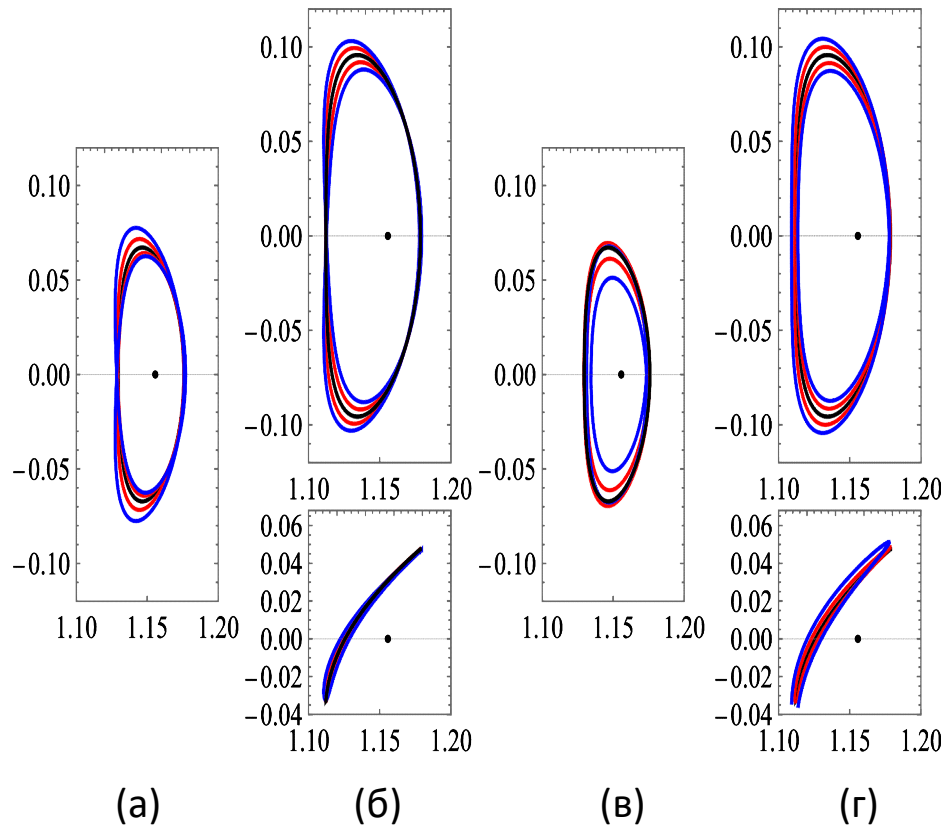


Fig. 21 Second order resonant orbits for the difference nominal acceleration value κ

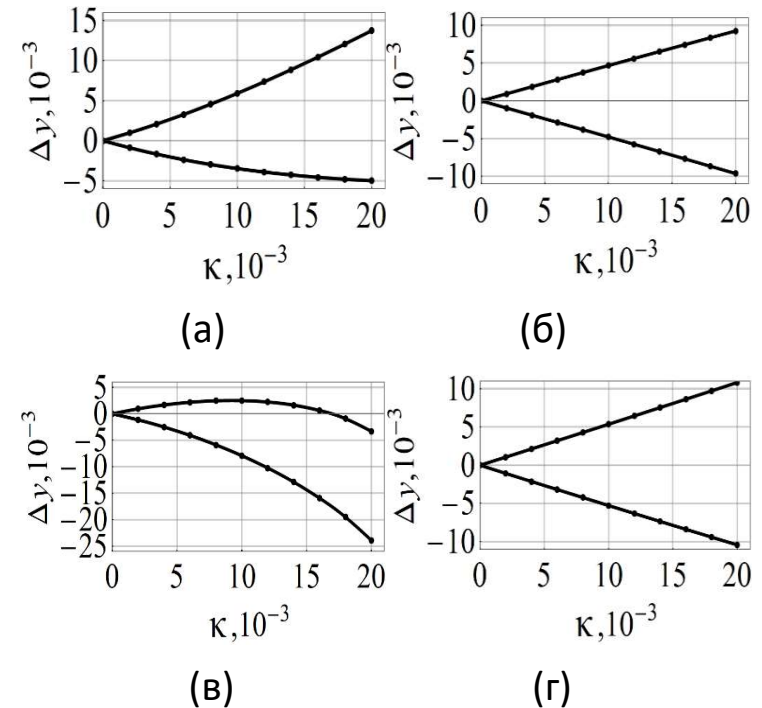


Fig. 22 The dependence of the maximum displacements in the y direction relative to natural orbits on the κ values

The nominal acceleration value uniquely determines the displacement of resonant orbits relative to natural.



3. Forming of the second order resonant orbits

3. . Influence of the installation angle α

$\alpha = 0^\circ$ $\alpha = 15^\circ$ $\alpha = 35.26^\circ$ $\alpha = 45^\circ$

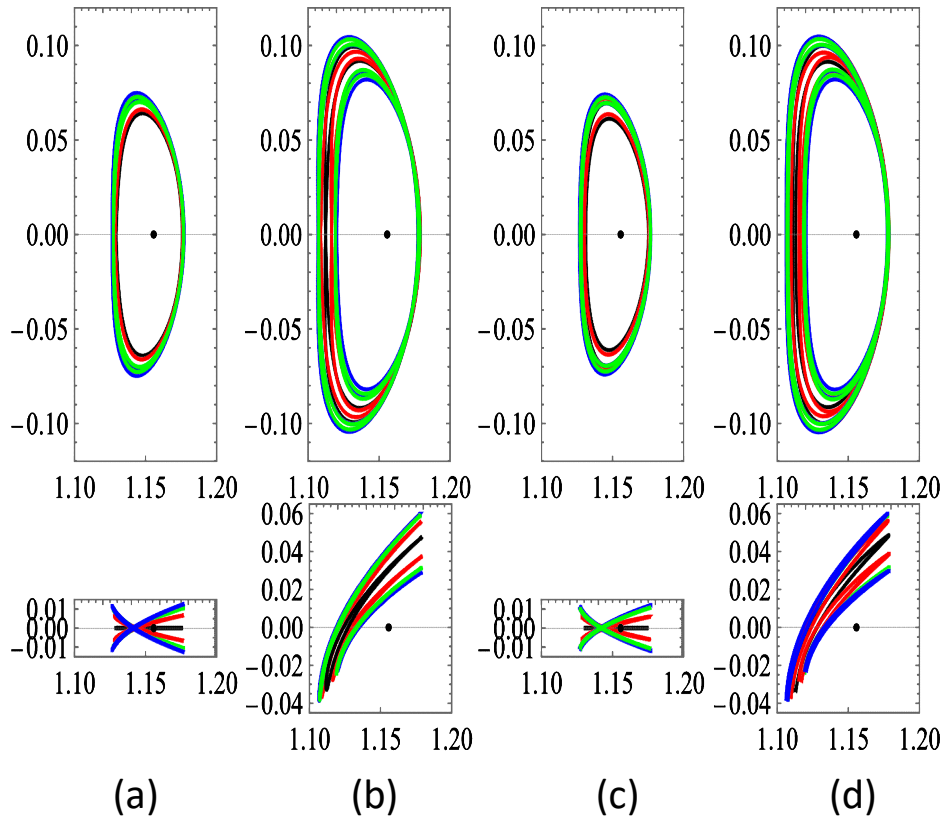


Fig. 22 Second order resonant orbits for the difference installation angle

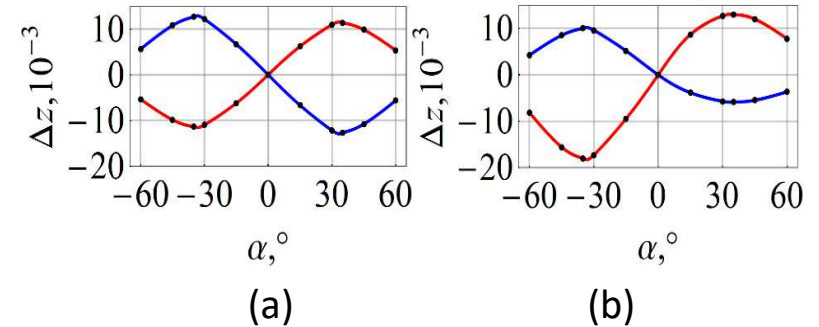


Fig. 22 Dependences of the maximum displacements in the z direction relative to natural orbits on the installation angle:
(a) for Lyapunov - conformal orbits;
(b) for halo - conformal orbits

The value of the installation angles determines the displacement of the in the z direction.

The maximum displacement are obtained at values of $\alpha = \pm 35,26^\circ$



3. Formation of artificial orbits. Cylindrical orbits

Cylindrical orbits are a type of orbit that are located entirely below or above the lunar orbital plane at a certain altitude (**Fig. 23**).

Methodology for forming cylindrical orbits: we need to find analytical solutions of linearized motion equations near the libration point.

Assumptions at the stage of orbits formation:

1. SS has an ideal reflective surface ($\mathbf{u} = \mathbf{0}$);
2. The SS installation angles are fixed;
3. It is believed that $\cos i = 1$, to make the change of the solar pressure vector in the $oxyz$ system periodic:

$$\begin{cases} a_x = \kappa \cos^2 \alpha (\cos \alpha \cos \lambda - \sin \alpha \sin \gamma \sin \lambda + \sin i \sin \alpha \cos \gamma \sin \theta) \\ a_y = \kappa \cos^2 \alpha (\cos \alpha \sin \lambda + \sin \alpha \sin \gamma \cos \lambda + \sin i \sin \alpha \cos \gamma \cos \theta) \\ a_z = \kappa \cos^2 \alpha [\sin i (-\cos \alpha \sin \varphi + \sin \alpha \sin \gamma \cos \varphi) + \sin \alpha \cos \gamma] \end{cases}$$

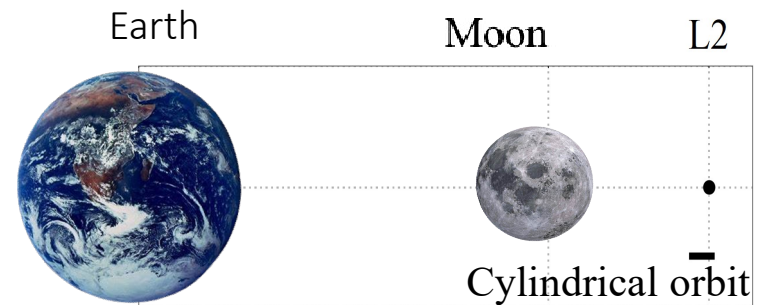


Fig. 23 Relative position of the Earth, Moon and the cylindrical orbit

Linearized equations of motion:

$$\begin{cases} \ddot{\xi} - 2\dot{\eta} = U_{xx}\xi + a_x \\ \ddot{\eta} + 2\dot{\xi} = U_{yy}\eta + a_y \\ \ddot{\zeta} = -U_{zz}\zeta + a_z \end{cases} \quad (5)$$



3. Formation of artificial orbits. Cylindrical orbits

Analytical solution:

$$\begin{cases} \xi(t) = \xi_0 \cos \lambda + \xi_1 \sin \lambda + \xi_2 \sin \theta \\ \eta(t) = \eta_0 \sin \lambda + \eta_1 \cos \lambda + \eta_2 \cos \theta \\ \zeta(t) = \zeta_0 + \zeta_1 \sin \varphi + \zeta_2 \cos \varphi \end{cases}$$

$\xi_0, \xi_1, \xi_2, \eta_0, \eta_1, \eta_2, \zeta_0, \zeta_1, \zeta_2$ are determined

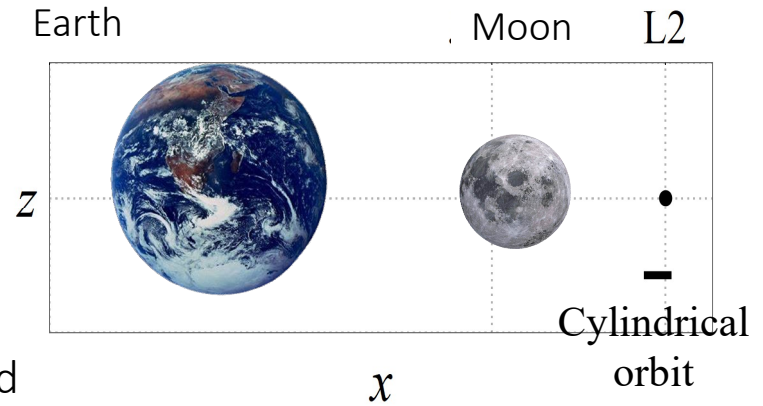


Fig. 23 Relative position of the Earth, Moon and the cylindrical orbit

$$\xi_0 = \frac{(\omega_C^2 + 2\omega_C + U_{yy})\kappa \cos^3 \alpha}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_C^2 - \omega_C^4}$$

$$\xi_1 = \frac{(\omega_C^2 + 2\omega_C + U_{yy})\kappa \cos^2 \alpha \sin \alpha \sin \gamma}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_C^2 - \omega_C^4}$$

$$\xi_2 = \frac{(\omega_M^2 + 2\omega_M + U_{yy})\kappa \cos^2 \alpha \sin \alpha \cos \gamma \sin i}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_M^2 - \omega_M^4}$$

$$\eta_0 = -\frac{(\omega_C^2 + 2\omega_C + U_{xx})\kappa \cos^3 \alpha}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_C^2 - \omega_C^4}$$

$$\eta_1 = -\frac{(\omega_C^2 + 2\omega_C + U_{xx})\kappa \cos^2 \alpha \sin \alpha \sin \gamma}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_C^2 - \omega_C^4}$$

$$\eta_2 = -\frac{(\omega_M^2 + 2\omega_M + U_{xx})\kappa \cos^2 \alpha \sin \alpha \cos \gamma \sin i}{-U_{xx}U_{yy} - (U_{xx} + U_{yy} - 4)\omega_M^2 - \omega_M^4}$$

$$\zeta_0 = -\frac{\kappa \cos^2 \alpha \sin \alpha \cos \gamma}{U_{zz}}$$

$$\zeta_1 = \frac{\kappa \cos^3 \alpha \sin i}{\omega_3^2 + U_{zz}}$$

$$\zeta_2 = -\frac{\kappa \cos^2 \alpha \sin \alpha \sin \gamma \sin i}{\omega_3^2 + U_{zz}}$$



3. Formation of artificial orbits. Cylindrical orbits

Conditions for obtaining cylindrical orbits with the maximum displacement value under the Moon's south pole $\alpha_0 \approx 35.26^\circ$, $\gamma_0 = 180^\circ$

1. Quasiperiodic cylindrical orbits :

$$\begin{cases} \xi(t) = \xi_0 \cos \lambda + \xi_2 \sin \theta \\ \eta(t) = \eta_0 \sin \lambda + \eta_2 \cos \theta \\ \zeta(t) = \zeta_0 + \zeta_1 \sin \varphi \end{cases} \quad (7)$$

Displacement : $H_K^* = |\zeta_0| - |\zeta_1| = 0,105\kappa$

2. Introducing an additional assumption $\sin i = 0$ periodic cylindrical orbits were obtained:

$$\begin{cases} \xi(t) = \xi_0 \cos \lambda \\ \eta(t) = \eta_0 \sin \lambda \\ \zeta(t) = \zeta_0 \end{cases} \quad (8)$$

Displacement : $H_\Pi^* = |\zeta_0| = 0,121\kappa$

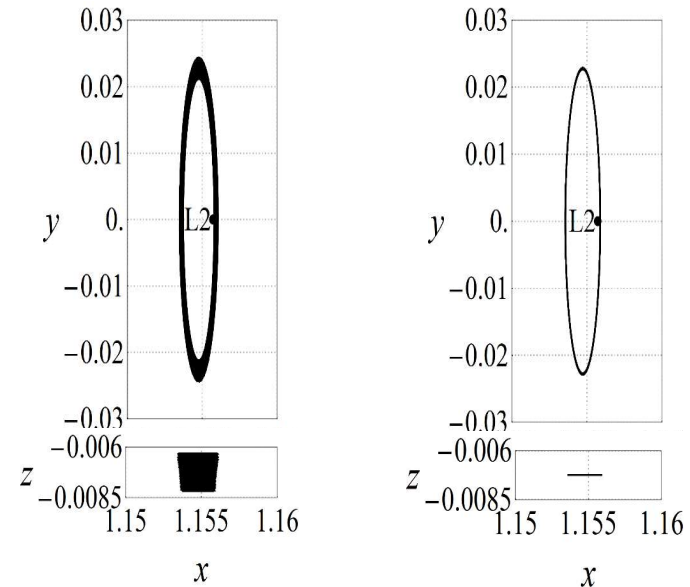


Fig. 24 Quasiperiodic (left) and periodic (right) cylindrical orbits under the Moon south pole

Table 2 Parameters of the quasi-periodic (QPCO) and periodic (PCO) cylindrical orbits*

Orbit	κ	α°	γ°	H^* , km	Period, days
QPCO	0.06	35.26	180	2429.14	29.53
PCO	0.06	35.26	180	2782.66	29.53



3. Maintenance of artificial orbits

Introduced assumptions and the linearization of the motion equations during the artificial periodic orbits formation lead to the presence of **unaccounted disturbing factors**.

$$\Delta \mathbf{a} = \ddot{\mathbf{r}}_0 + 2\boldsymbol{\omega}_{\text{JI}} \times \dot{\mathbf{r}}_0 - \nabla U(\mathbf{r}_0) - \mathbf{a}_{\text{CB}}$$

$\mathbf{r}_0$ – position vector on an artificial periodic orbit without taking into account disturbances.

The technique of maintaining artificial orbits is a sliding mode control method: the control variables are changed so as to make the system move along a sliding section.

Computational procedure for determining control for maintaining artificial orbits :

1. Sliding surface : $\mathbf{S} = \mathbf{r} - \mathbf{r}_0 + \boldsymbol{\Gamma}(\dot{\mathbf{r}} - \dot{\mathbf{r}}_0)$

$\boldsymbol{\Gamma}$ – positive definite velocity deviation weight matrix

2. Lyapunov function: $V = \frac{1}{2} \mathbf{S}^T \mathbf{S}$

3. Control law: $\mathbf{a}_{\text{yB}} = \ddot{\mathbf{r}}_0 + 2\boldsymbol{\omega}_{\text{JI}} \times \dot{\mathbf{r}} - \nabla U(\mathbf{r}) - \boldsymbol{\Gamma}^{-1}(\boldsymbol{\sigma} \mathbf{S} + \dot{\mathbf{r}} - \dot{\mathbf{r}}_0) \quad (9)$

As a result, $\dot{V} = -\mathbf{S}^T \boldsymbol{\sigma} \mathbf{S} < 0$. The system is asymptotically stable.



3. Maintenance of cylindrical artificial orbits

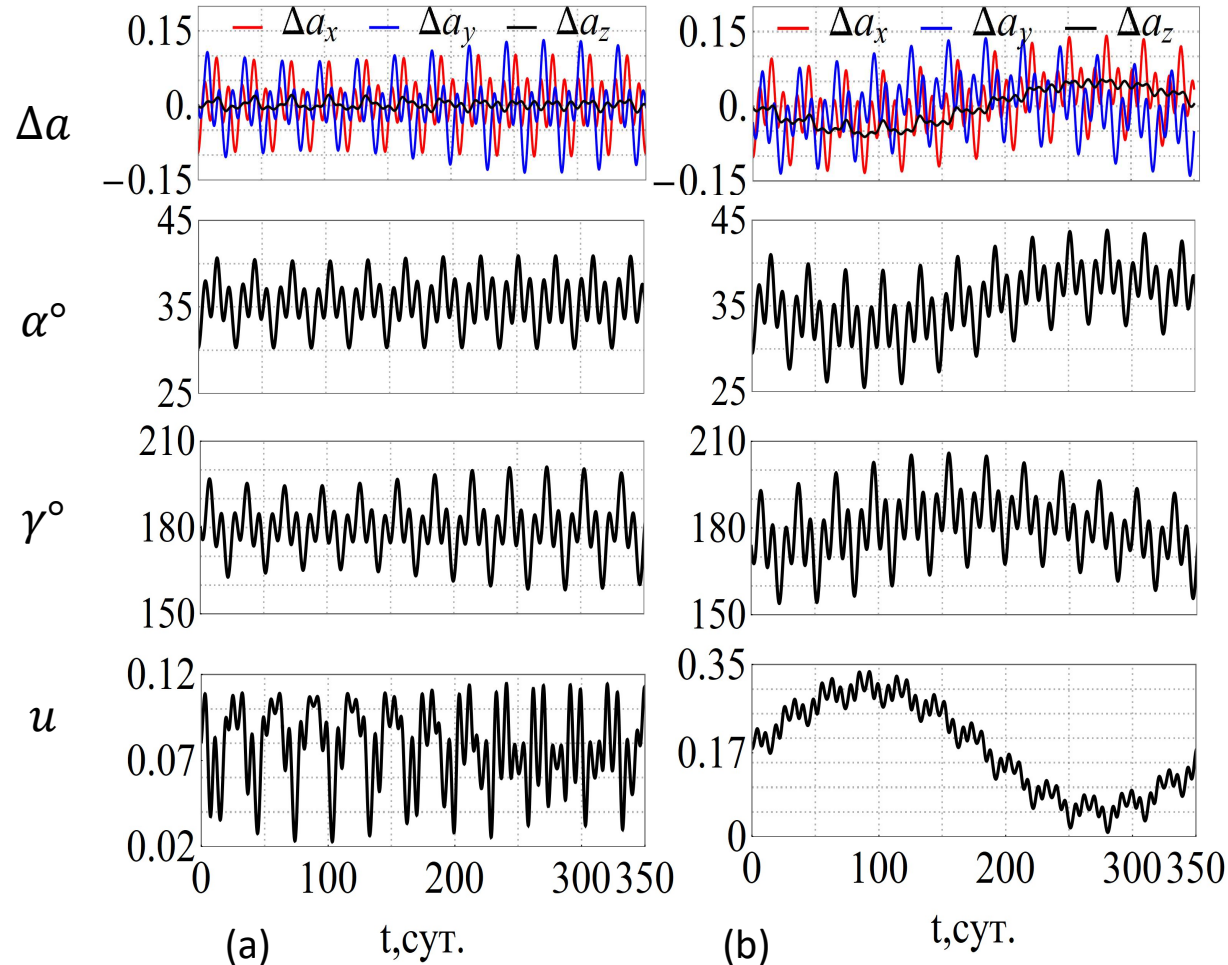


Fig. 25 Changes in projections of disturbing accelerations and SS control variables to maintenance orbits :
(a) Cylindrical quasi-periodic ($\alpha_0 = 35,26^\circ, \gamma_0 = 180^\circ, u_0 = 0,1$);
(b) Cylindrical periodic ($\alpha_0 = 35,26^\circ, \gamma_0 = 180^\circ, u_0 = 0,2$).



3. Maintenance of resonant artificial orbits

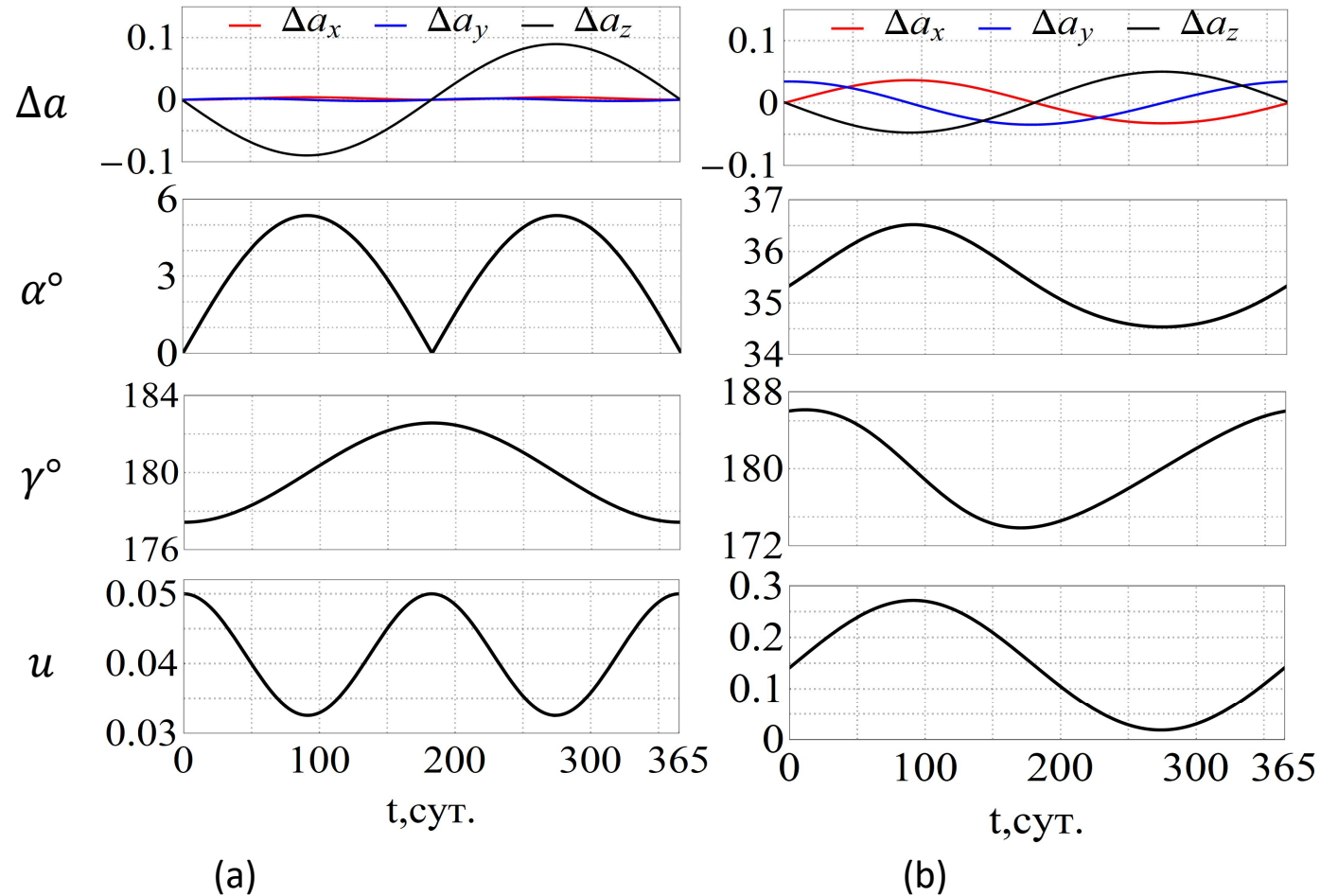


Fig. 26 Changes in projections of disturbing accelerations and SS control variables to maintain orbits :
(a) Resonant ($\alpha_0 = 0^\circ, \gamma_0 = 180^\circ, u_0 = 0.15$); (b) Resonant ($\alpha_0 = 35.26^\circ, \gamma_0 = 180^\circ, u_0 = 0.05$);



4. Control of launching SSSC into artificial orbits

1. Optimization problem with the criterion - maximum accuracy of inference (max-A)

Objective function :

$$J_{\text{max-A}} = \frac{1}{2} \left(\mathbf{D} \left[\mathbf{X}(t_f) - \mathbf{X}_f(t_f) \right] \right)^2 \quad (11)$$

$\mathbf{X}_f(t)$ – state vector in the target orbit

t_f – final moment of flight

$$\mathbf{D} = \text{diag}(3700, 3700, 3700, 1, 1, 1)$$

– weight matrix (the values are chosen so that a position deviation of 1 m has the same weight as a velocity deviation of 1 cm/s)

According to Pontryagin's maximum principle

Equations of conjugate variables:

$$\dot{\lambda}_r = -\frac{\partial \mathbf{g}^T(\mathbf{r})}{\partial \mathbf{r}} \lambda_v, \quad \dot{\lambda}_v = -\lambda_r - \frac{\partial \mathbf{h}^T(\mathbf{v})}{\partial \mathbf{v}} \lambda_v \quad (12)$$

Hamilton function:

$$H = \lambda_r^T \mathbf{v} + \lambda_v^T \left[\mathbf{g}(\mathbf{r}) + \mathbf{h}(\mathbf{v}) + \mathbf{a}_{\text{CB}}(\boldsymbol{\alpha}, t) \right] \quad (13)$$

Boundary and transversality conditions:

$$\mathbf{X}(t_0) = \mathbf{X}_0, \quad \begin{bmatrix} \lambda_r(t_f) \\ \lambda_v(t_f) \end{bmatrix}^T = \mathbf{D} \left[\mathbf{X}(t_f) - \mathbf{X}_f(t_f) \right] \quad (14)$$

Optimal control variables for the SS:

$$\alpha^* = \alpha \left| \min \lambda_v^T \mathbf{a}_{\text{CB}}(\boldsymbol{\alpha}, t), \gamma^* = \tan^{-1} \frac{\lambda_{v2}}{\lambda_{v3}} \right.$$

$$u^* = \frac{u_{\text{max}} - u_{\text{min}}}{2} \text{sign}(S) + \frac{u_{\text{max}} + u_{\text{min}}}{2} \quad (15)$$

$$S = -\frac{\lambda_{v1}}{2} \cos \alpha \cos 2\alpha - \sin \alpha \cos^2 \alpha \left(\lambda_{v2} \sin \gamma^* + \lambda_{v3} \cos \gamma^* \right)$$

$u_{\text{max}} = 0,4$ and $u_{\text{min}} = 0$ – maximum and minimum values of the reflection coefficient determined by the SS design



4. Control of launching SSSC into artificial orbits

2. An optimization problem with the criterion being the minimum flight duration (min-T).

Objective function : $J_{\text{мин-ВЛ}} = \int_{t_0}^{t_f} 1 \, dt \quad (16)$

Hamilton function: $H = \lambda_r^T \mathbf{v} + \lambda_v^T [\mathbf{g}(\mathbf{r}) + \mathbf{h}(\mathbf{v}) + \mathbf{a}_{\text{CB}}(\boldsymbol{\alpha}, t)] - 1 \quad (18)$

Boundary and transversality conditions:

$$\begin{aligned} \mathbf{X}(t_0) &= \mathbf{X}_0, \mathbf{X}(t_f) = \mathbf{X}_f(t_f) \\ H(t_f) &= -\lambda_r^T(t_f) \dot{\mathbf{r}}_f(t_f) + \lambda_v^T(t_f) \dot{\mathbf{v}}_f(t_f) \end{aligned} \quad (19)$$

Optimal control variables for the SS:

$$\begin{aligned} \alpha^* &= \alpha \Big| \min \lambda_v^T \mathbf{a}_{\text{CB}}(\boldsymbol{\alpha}, t), \gamma^* = \tan^{-1} \frac{\lambda_{v2}}{\lambda_{v3}} \\ u^* &= \frac{u_{\text{мак}} - u_{\text{мин}}}{2} \text{sign}(S) + \frac{u_{\text{мак}} + u_{\text{мин}}}{2} \\ S &= -\frac{\lambda_{v1}}{2} \cos \alpha \cos 2\alpha - \sin \alpha \cos^2 \alpha (\lambda_{v2} \sin \gamma^* + \lambda_{v3} \cos \gamma^*) \end{aligned} \quad (20)$$



4. Control of launching SSSC into artificial orbits

The method for solving the optimization problem (see Fig. 27) uses the collocation method to solve optimization problems;

The continuation method by parameter is used to transform the natural orbit into the optimal trajectory of the SSSC, using the max-A criterion:

$$\kappa = \kappa + \Delta\kappa$$

$$t_f = t_f + \Delta t$$

The optimal transfer trajectory with the goal of max-A is used as an initial approximation for solving the problem of optimal time (min-T).

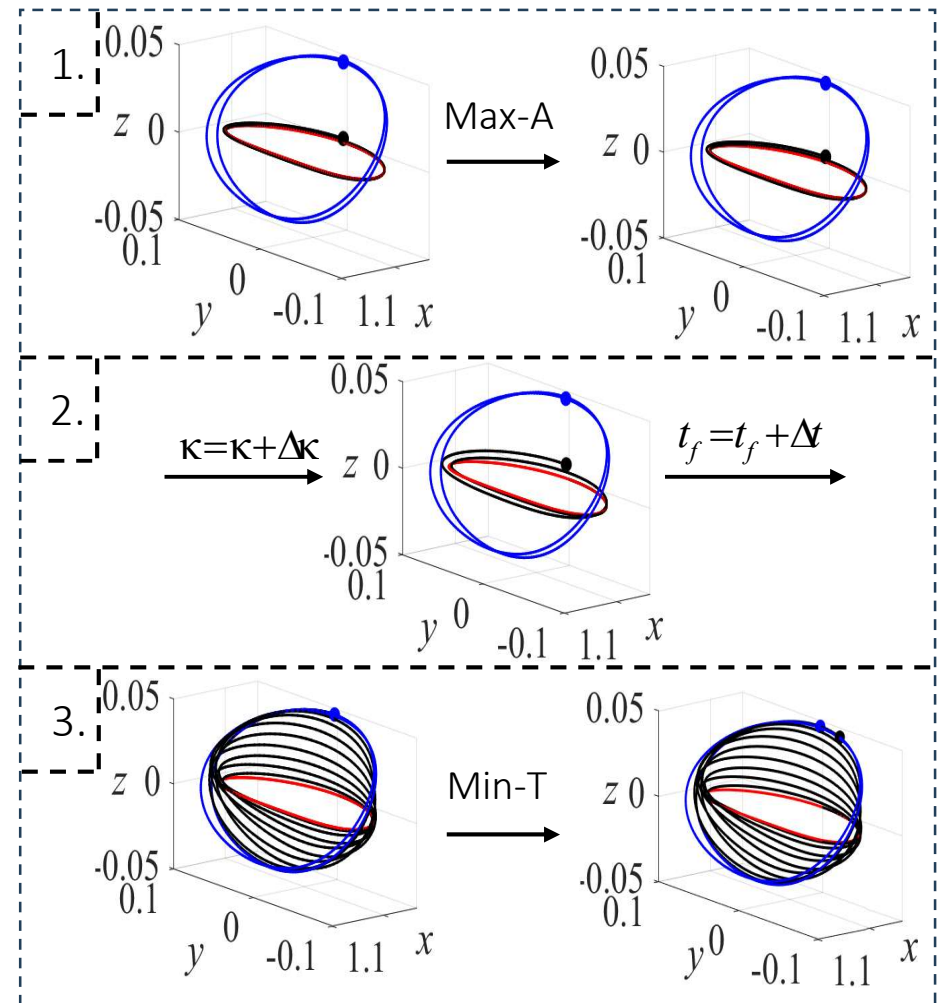


Fig. 27 Scheme of the computational procedure for solving the problem of optimizing the control of a transfer to an artificial orbit



4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into resonant orbits

1. The influence of the reference orbit energy on the minimum launch duration.

The energy of the spacecraft's state and orbit is determined by the Jacobi integral

Reference orbits: halo orbits with the different Jacobi integral values

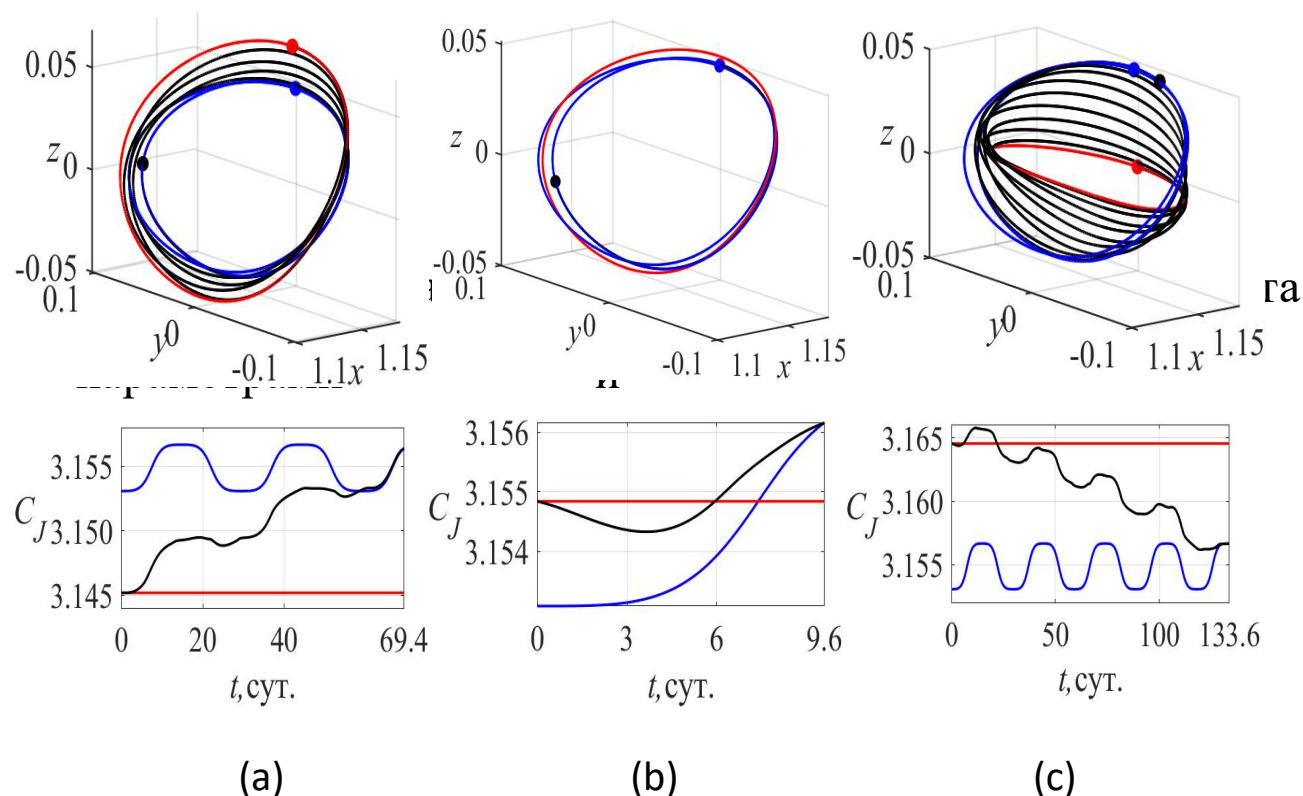


Fig. 28 Flight trajectories and changes in the Jacobi integral for different values of the Jacobi integral:

(a) $C_J = 3.145$; (b) $C_J = 3.155$; (c) $C_J = 3.164$.

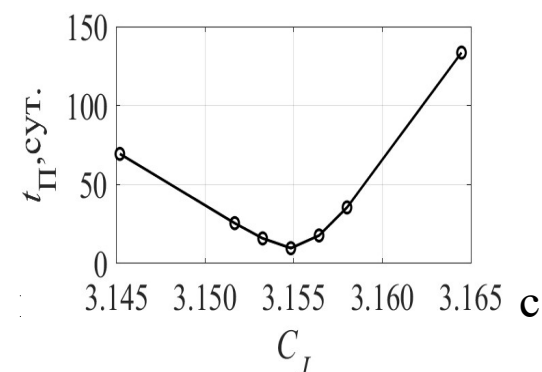


Fig. 29 The dependence of the flight time on the Jacobi integral of the reference orbits ($\kappa=0.008$; $\alpha_0=0^\circ$)

The minimum flight time is achieved when the Jacobi integral on the reference orbit equals to the average value on the target orbit



4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into resonant orbits

2. The effect of the phase shift difference on the minimum flight time.

The phase difference is determined by the time difference Δt_0 between CS and the target point, which passes through the phase angle $\vartheta=0^\circ$

Reference orbits: halo orbits with $C_J=3,155$;

Target orbit: halo-conformal resonant orbit of the first configuration.

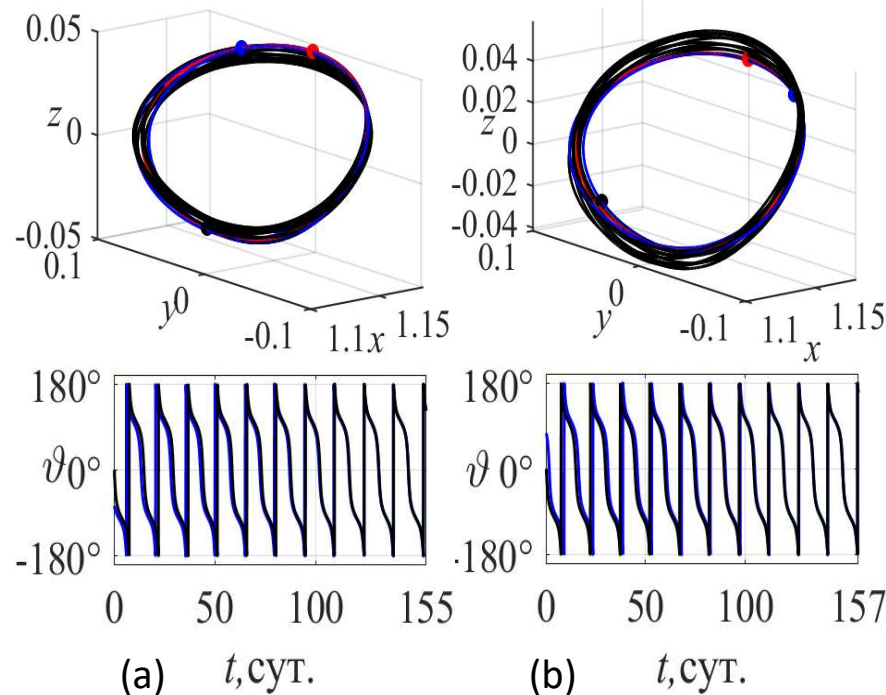


Fig. 30 Transfer trajectories and changes in phase position for different phase positions:

(a) $\Delta t_0 = 0.05 T_c$; (b) $\Delta t_0 = -0.05 T_c$

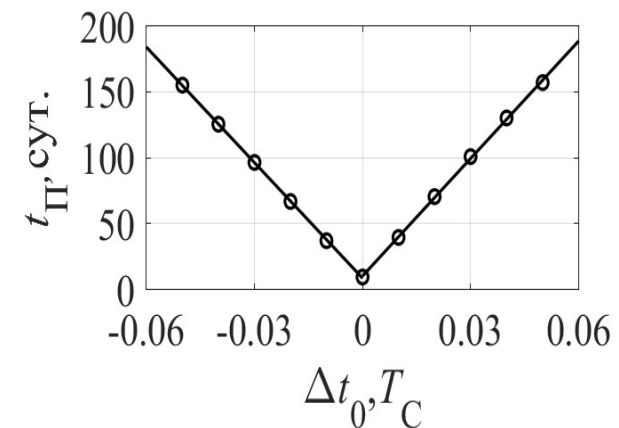


Fig. 31 The dependence of flight time t_{Π} on the difference in phase position Δt_0 between SC and the target point

The minimum flight time is achieved when the phase difference between SC and the target point is zero



4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into resonant orbits

3. The influence of the initial phase SC position on the minimum flight time.

The initial phase SC position is determined by the time t_0 elapsed after the phase angle $\vartheta=0$.

Reference orbits: halo orbits with $C_J=3,155$;

Target orbit: halo-conformal resonant orbit of the first configuration.

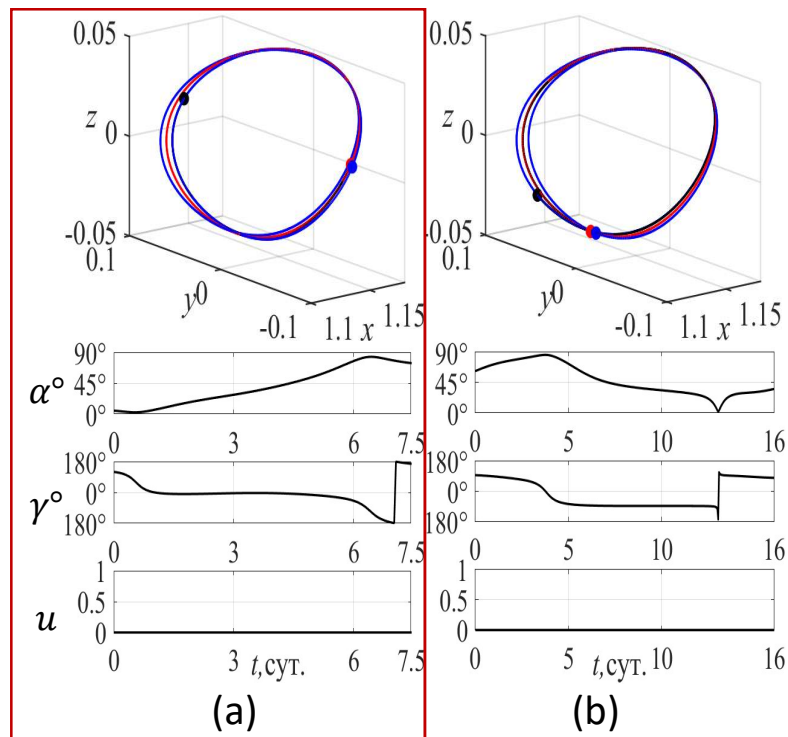


Fig. 32 Flight trajectories and changes in the control variables under different initial phase SC positions (a) $t_0 = 0,125 T_c$, (b) $t_0 = 0,25 T_c$

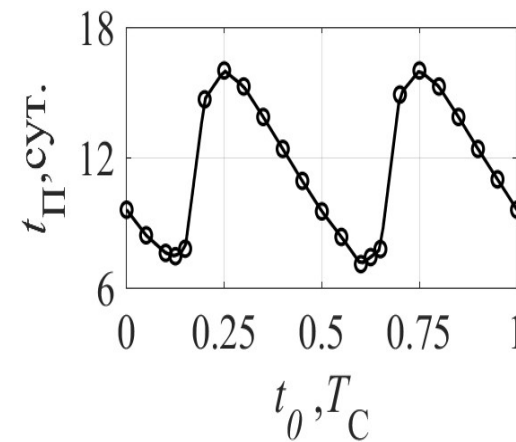


Fig. 33 The dependence of the flight time t_{Π} on the initial phase SC position t_0

The minimum flight duration is achieved when the initial phase SC position equals $t_0 = 0,125 T_c$, or $t_0 = 0,6 T_c$.



4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into cylindrical orbits

1. The influence of the reference orbit energy on the minimum launch duration.

Reference orbits: Lyapunov orbits with different values of the Jacobi integral.

Target orbit: periodic cylindrical orbit with parameters $\kappa=0,06$; $\alpha=35,26^\circ$, and $\gamma=180^\circ$

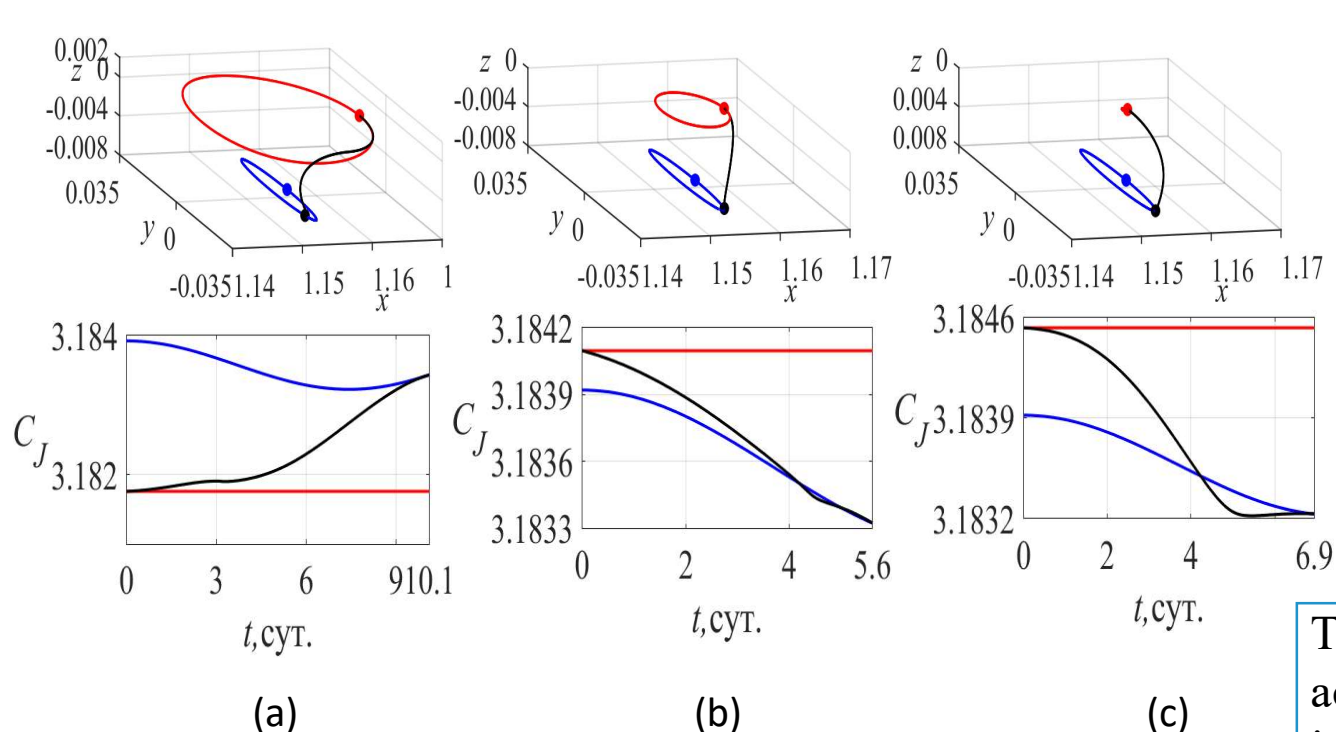


Fig. 34 Flight trajectories and changes in the Jacobi integral for different values of the Jacobi integral:

(a) $C_J = 3,18176$; (b) $C_J = 3,1840$, (c) $C_J = 3,18453$

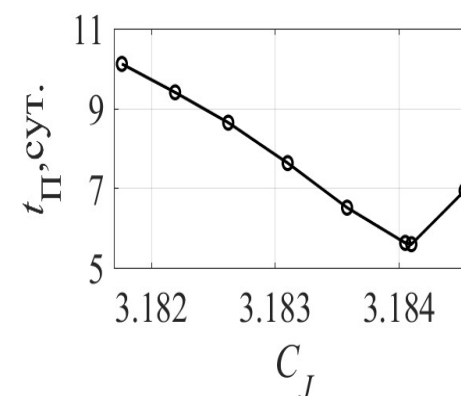


Fig. 35 The dependence of the flight time on the Jacobi integral of the reference orbits ($\kappa=0,008$; $\alpha_0=0^\circ$)

The minimum flight time is achieved when the Jacobi integral on the reference orbit, $C_J = 3.18453$, is greater than the average on the target orbit, $C_J = 0.1841$.

4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into cylindrical orbits

2. The effect of the phase shift difference on the minimum flight time.

Reference orbits: halo orbits with $C_J=3,1841$;

Target orbit: periodic cylindrical orbit with parameters $\kappa=0,06$; $\alpha_0=36,26^\circ$, $\gamma_0=180^\circ$

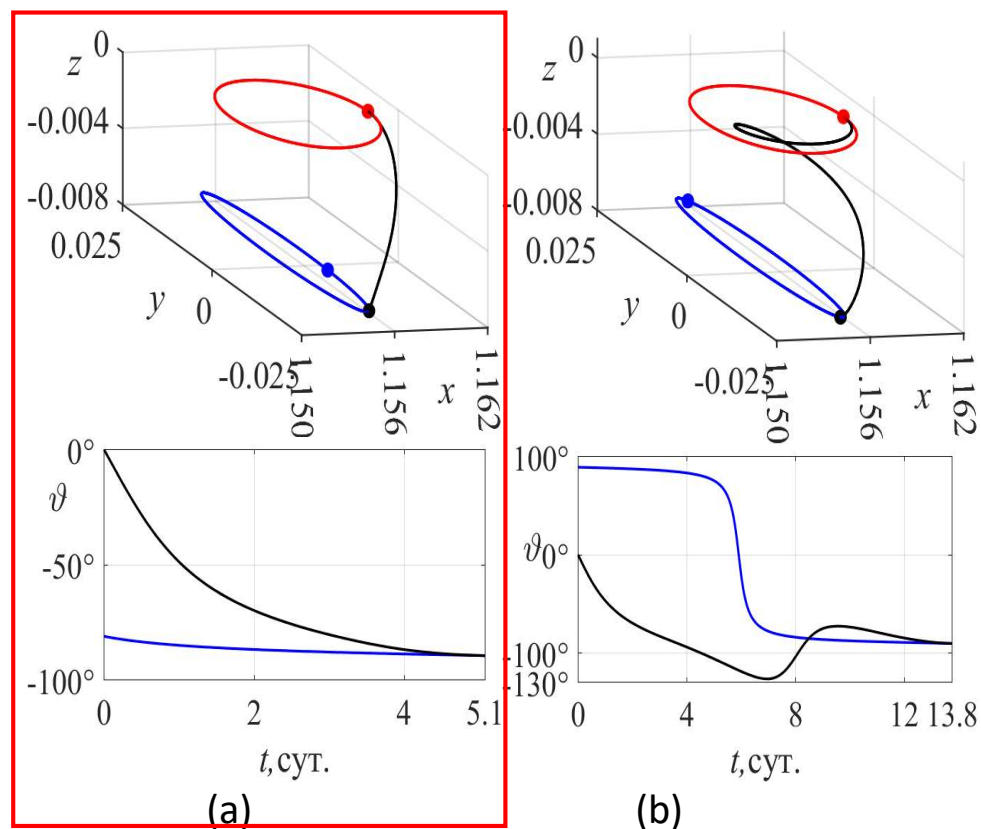


Fig. 36 Transfer trajectories and changes in phase position for different phase SC positions:

(a) $\Delta t_0 = 0,05 T_c$; (b) $\Delta t_0 = -0,2 T_c$

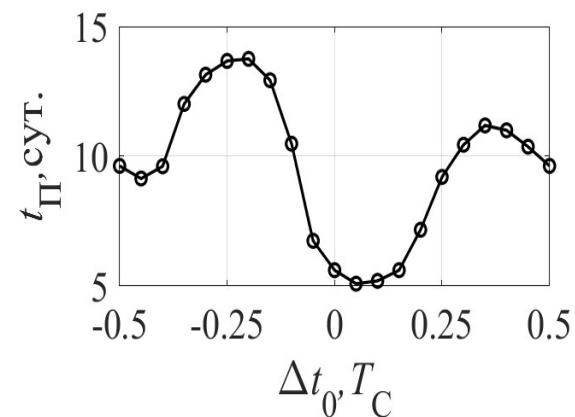


Fig. 37 The dependence of flight time t_{Π} on the difference in phase position Δt_0 between SC and the target point

The minimum flight time is achieved when the phase difference between SC and the target point $\Delta t_0 = 0,05 T_c$

4. Control of launching SSSC into artificial orbits

Analysis of solutions to the problem of injecting into cylindrical orbits

3. The influence of the initial phase SC position on the minimum flight time.

Reference orbits: halo orbits with $C_J=3,1841$; $\Delta t_0 = 0,05 T_c$;

Target orbit: periodic cylindrical orbit with parameters $\kappa=0,06$; $\alpha_0=36,26^\circ$, $\gamma_0=180^\circ$

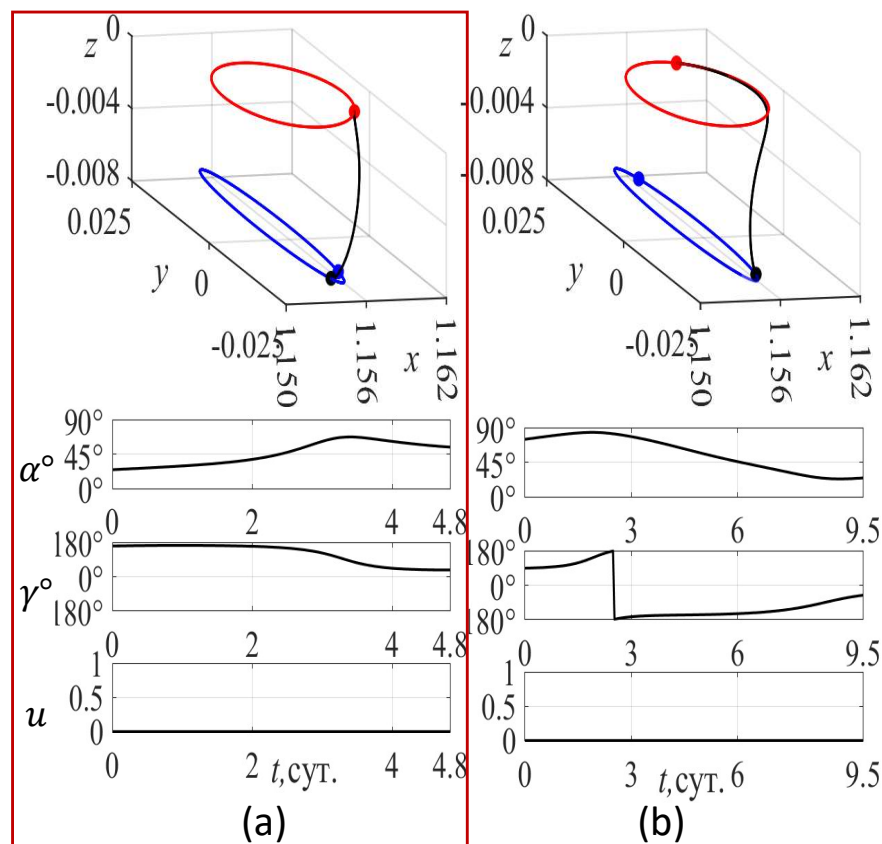


Fig. 38 Flight trajectories and changes in the control variables under different initial phase SC positions (a) $t_0=0,6T_c$, (b) $t_0=0,3T_c$

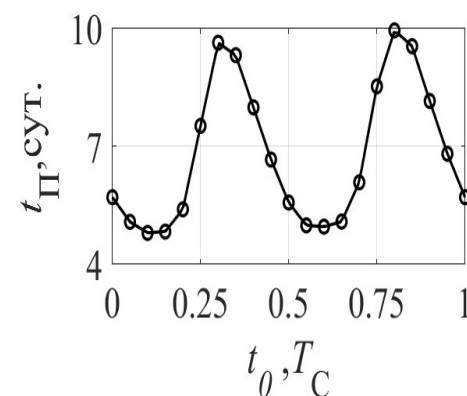


Fig. 39 The dependence of the flight time t_{Π} on the initial phase SC position t_0

The minimum flight duration is achieved when the initial phase SC position equals $t_0=0,6 T_c$.



Main results

1. **Methods** have been developed for **forming resonant and cylindrical orbits** for spacecraft with SS in the vicinity of the L2 point of the Earth-Moon system.
2. The **dependences of the geometric characteristics** of artificial orbits on the SS parameters and control variables are analyzed. By combining different parameter values, we can obtain a variety of orbits that can be adapted to different mission needs..
3. A **method for maintaining artificial orbits** using SS control has been developed, taking into account the peculiarities of the movement of the Sun-Earth-Moon system. It is shown that by changing the control parameters it is possible to compensate influence of previously unaccounted factors. This ensures the possibility of long-term operation of the SS spacecraft in such orbits.
4. A technique for **forming optimal control** of spacecraft launch from natural orbits to artificial orbits has been developed. This solved the problems of low computational efficiency and poor adaptability of existing methods.
5. The **dependences of the minimum duration** of launching into artificial orbits on the choice of reference orbits and the initial position of the spacecraft are analyzed.

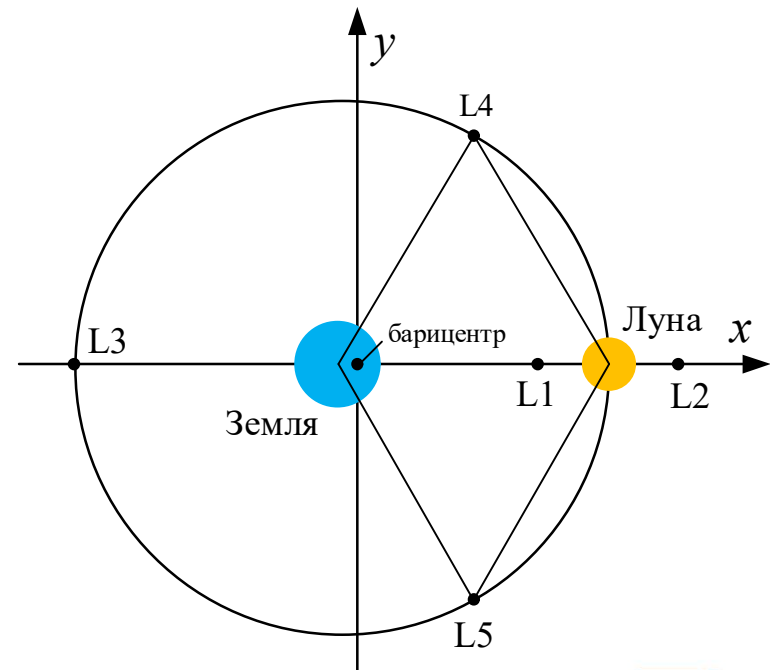


◆ DISTURBANCES

Circular restricted three body problem

- 1) the orbit of motion of a small celestial object is circular
- 2) the system does not take into account perturbing factors.

For example, the influence of the **Sun's gravity** cannot be neglected for the motion of the spacecraft in circumlunar space. Second, the **eccentricity** of the Moon's orbit around the Earth varies between 0.0255 and 0.0775, and the plane of the orbit is not fixed in space due to lunar precession and wobble.





◆ DISTURBANCES

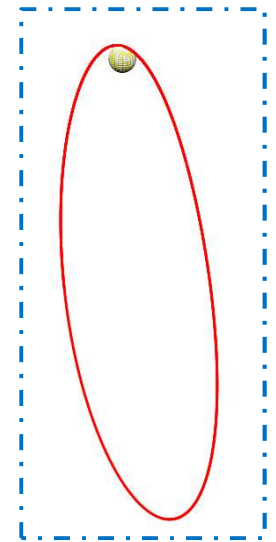
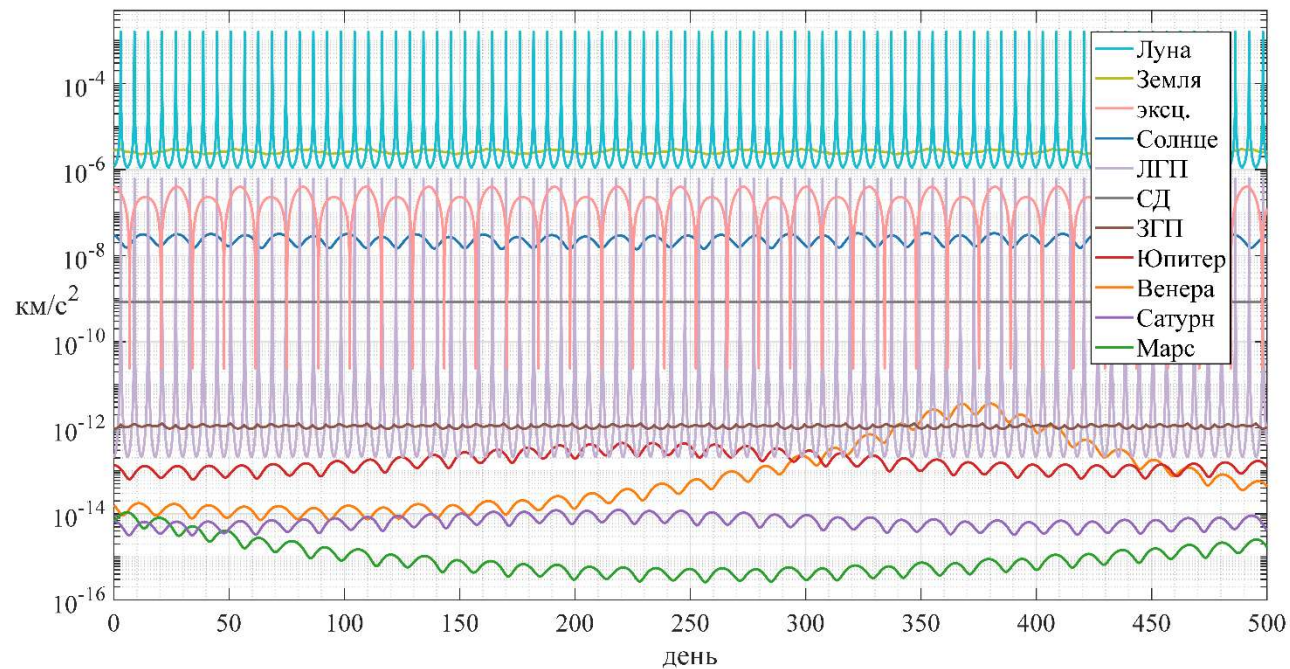
Disturbance from the **gravity** of the solar system

Disturbance from a **non-central gravitational field** of Earth and Moon

Influence of peculiarities of the **Moon's movement**

Disturbance from **solar pressure**

Effect of **eccentricity** of the Moon's orbit around the Earth





◆ DISTURBANCES

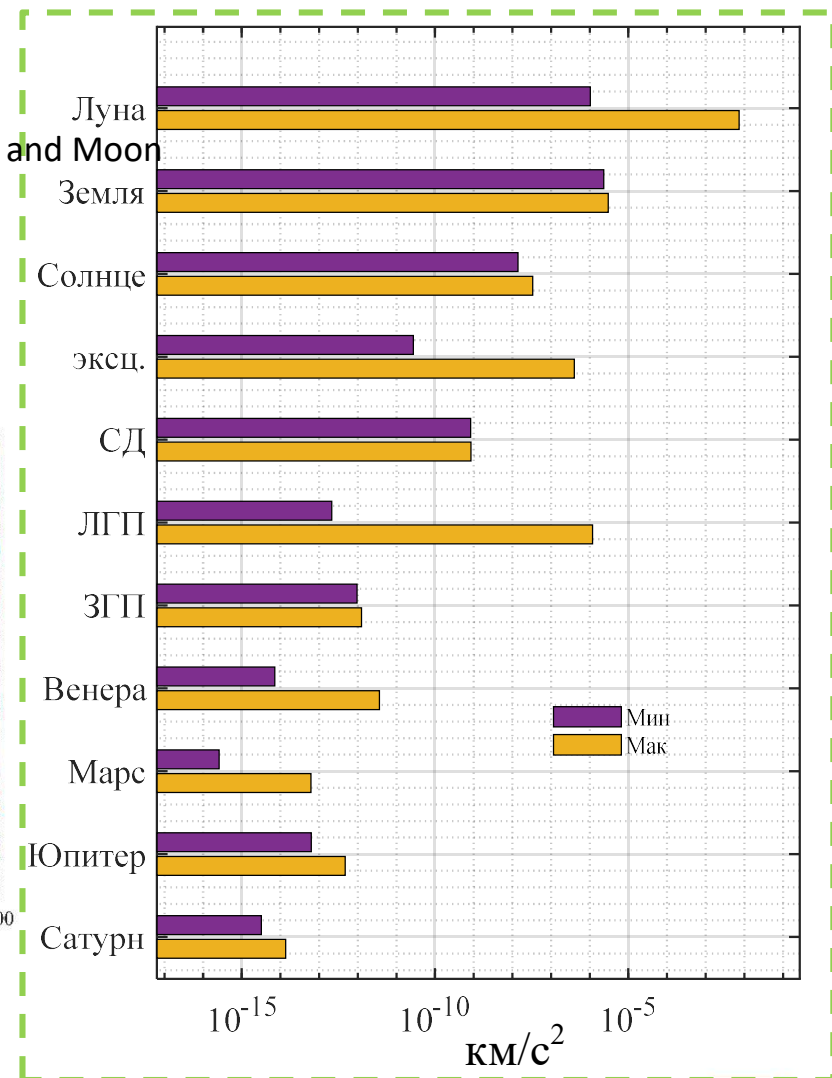
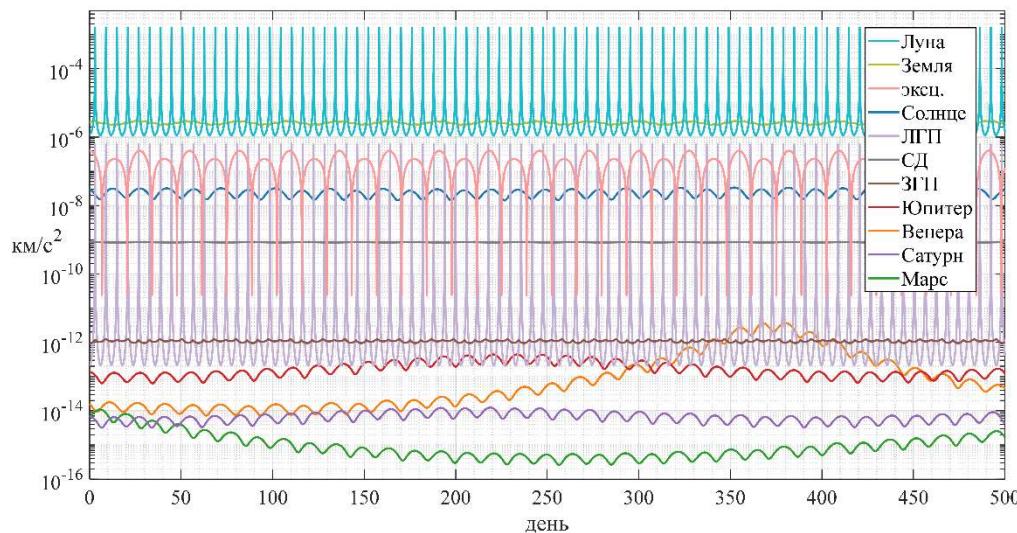
Disturbance from the **gravity** of the solar system

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Influence of peculiarities of the **Moon's movement**

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Effect of **eccentricity** of the Moon's orbit around the Earth





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