Artificial Periodic Orbits of a Solar Sail Spacecraft in Moon Vicinity

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Abstract. In the framework of future projects for the exploration and development of the Moon, relay satellites for communication between the Earth and the Moon—especially between the Earth and the far side and polar regions of the Moon, where permanent bases are planned to be built—are of significant interest. This article develops a methodology for determining resonant orbits near the L2 libration point in the Earth-Moon system based on the model of the circular restricted three-body problem. In addition, the influence of changes in the initial position of the Sun, the magnitude of nominal acceleration from light pressure, and the installation angles of the solar sail on such artificial resonant orbits is considered. As a result of the study, new types of periodic orbits for placing relay satellites in the Earth-Moon system, and the dependencies of the shape and size of artificial resonant orbits on design parameters and control parameters of spacecraft with a solar sail, were obtained.

Keywords. Resonant orbit, solar sail, L2 libration point, Earth-Moon system, multiple shooting method, initial position of the Sun.

Introduction

In the near future, the exploration and use of the Moon will become the main task of space exploration. The far side and polar regions of the Moon are of particular interest for development. The functioning of objects on the Lunar surface in these areas will require the creation of satellite systems to ensure continuous communication between the Earth and the Moon. Usually, such satellites are placed in natural periodic orbits (for example, Lyapunov orbits and halo orbits [1, 2]) near the L2 point. However, for some tasks, it is required to simultaneously meet the requirements for the period, the shape, and the position of the orbit.

Solar sails (SS) represent a relatively new technology for creating control action in space due to light pressure [3]. The key advantage of an SS is the absence

of fuel consumption. Bookless and McInnes [4] investigated a method for forming artificial Lagrange points in the Sun-Earth system created by shifting natural libration points due to solar light pressure. Changing the parameters and control laws of the SS allows obtaining various artificial Lagrange points or forming artificial periodic orbits in accordance with specific requirements.

We have developed a new methodology for determining the resonant orbits of an SS near the L2 point in the Earth-Moon system, corresponding to given sail installation angles, the magnitude of the nominal SS acceleration, and the initial position of the Sun.

1. Mathematical Models and Methods

If we neglect all perturbations, the mass of the spacecraft, and consider circular trajectories of the Earth and the Moon around the Sun and the Earth respectively, then the motion of the spacecraft in the coordinate system **oxyz** can be considered within the framework of the circular restricted three-body problem model. In this case, the equation of spacecraft motion in the rotating coordinate system has the form:

$$\ddot{\mathbf{r}} + 2\omega_M \times \dot{\mathbf{r}} + \nabla U(\mathbf{r}) = \mathbf{a}_s, \tag{1}$$

where $\mathbf{r}$ is the spacecraft position vector; ω_M is the angular velocity of the Moon around the Earth; $\mathbf{a}_s$ is the acceleration from light pressure forces; $U(\mathbf{r})$ is the pseudo-potential function.

If we assume that the inclination of the Moon's orbit $i_M \approx 0$ is small, then subsequent analysis is significantly simplified. The rotating coordinate system the direction vector of sunlight $\mathbf{s}$ has the coordinates

$$\mathbf{s} = (\cos(\omega_s t + \theta_0),\ -\sin(\omega_s t + \theta_0),\ 0)^T, \tag{2}$$

where θ_0 is the initial position of the Sun; ω_s is the angular velocity of the Sun's rotation around the Earth-Moon system in the rotating coordinate system.

Assuming that the SS has a perfectly reflecting surface, the acceleration vector acting on the spacecraft $\mathbf{a}_s$ can be expressed as:

$$\mathbf{a}_s = \kappa(\mathbf{s} \cdot \mathbf{n})^2 \mathbf{n}, \tag{3}$$

where κ is the maximum (nominal) acceleration from light pressure at a given distance from the Sun; $\mathbf{n}$ is the unit normal vector to the SS. Let the vector $\mathbf{n}$ in the coordinate system be defined by the orientation angles α and δ:

$$\mathbf{n} = (\cos\alpha\cos\delta,\ \sin\alpha\cos\delta,\ \sin\delta)^T, \tag{4}$$

where α is the angle between the vector $\mathbf{n}$ and the plane; δ is the angle of rotation of the vector $\mathbf{n}$ around the axis.

The nominal acceleration from the SS κ and the angle δ are constant and the acceleration vector $\mathbf{a}_s$ from the light pressure forces is equal to

$$\mathbf{a}_s = \kappa\cos^2\alpha_0\cos^2\delta\mathbf{n}. \tag{5}$$

Analysis of equation (5) shows that the influence of the angle δ on the acceleration vector $\mathbf{a}_s$ can be considered equivalent to a change in the magnitude κ and the angle α.

In this work, to determine the resonant orbit corresponding to fixed installation angles, nominal SS acceleration, and the initial position of the Sun, a two-point boundary value problem is solved, in which the initial vector of the spacecraft's phase coordinates is determined, ensuring the closure of the spacecraft's flight trajectory over a period T. Natural orbits with a period equal to T are used as an initial approximation.

2. Simulation results

Let us consider the influence of the initial position of the Sun θ_0, the magnitude of nominal acceleration κ and the SS installation angle α on these orbits.

1. For the entire possible range of the Sun's initial position, only two configurations of resonant orbits can be obtained. They correspond to $\theta_0 = 0°$ or $180°$, and $\theta_0 = 90°$ or $270°$. Resonant orbits of the first configuration ($\theta_0 = 0°$ or $180°$) have two identical orbital loops, symmetrical relative to y. Resonant orbits of the second configuration ($\theta_0 = 90°$ or $270°$) have one large loop and one small loop, symmetrical to themselves relative to x.

2. The magnitude of nominal acceleration κ determines the displacement of the resonant orbits relative to the natural periodic orbits. Figure 1 presents the dependencies of the maximum displacements of the positions of resonant orbits Δy in the direction of the axis y relative to the natural periodic orbits on the magnitudes of the nominal acceleration κ. The upper line corresponds to the maximum displacement of the outer loop of the orbits, and the lower line corresponds to the maximum displacement of the inner loop of the orbits.

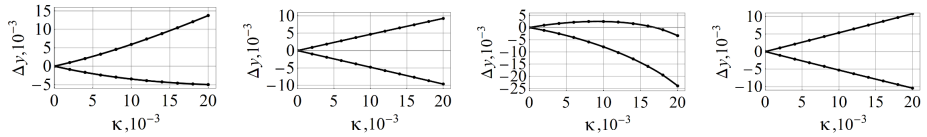
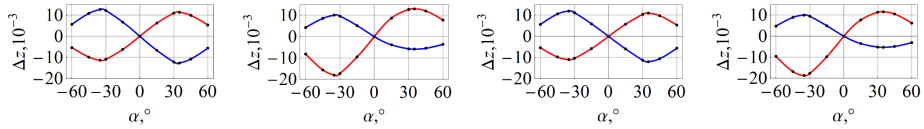


FIGURE 1. Dependencies of the maximum position displacements of resonant orbits Δz in the direction relative to natural periodic orbits on the inclination angle α.

3. Figure 2 presents the dependencies of the maximum position displacements of the resonant orbits relative to the natural periodic orbits on the sail installation angle α. The red lines represent the displacements of the highest points of the orbits, and the blue lines represent the displacement of the lowest points of the orbits.



(a) (b) (c) (d)

FIGURE 2. Dependencies of the maximum position displacements of resonant orbits Δz in the direction relative to natural periodic orbits on the inclination angle α.

Conclusion

In this work, a methodology for determining resonant periodic orbits relative to the L2 libration point of the Earth-Moon system is proposed, and it is determined how their configuration and displacement change compared to natural halo orbits when the initial position of the Sun, the magnitude of the nominal acceleration, and the SS installation angle change. By combining different values of the control variables, it is possible to create diverse orbits adaptable to various mission needs.

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