

A theory of secular orbital evolution of a binary black hole in a non-spherical stellar cluster

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Abstract. In this study we propose a new solution to the so-called "central parsec problem" of the orbital evolution of a supermassive binary black hole at scales of one parsec or less. At these scales, the standard mechanism of orbital decay driven by dynamical friction is inefficient. We discuss the orbital evolution through the formation of highly elongated orbits due to the action of a non-spherical part of gravitational potential of the central stellar cluster followed by their circularisation through the emission of gravitational waves. The non-spherical part of gravitation potential is assumed to be small and mainly determined by distribution of the stars at distances much larger than the orbital semi-major axis. In this approximation it has two non-trivial parts with azimuthal numbers $m = 0$ and 2 . While the former part leads to the well studied orbital dynamics of von Zeipel-Lidov-Kozai type, the latter causes a non-trivial secular evolution, which is studied analytically and numerically.

Introduction

The formation of gravitationally bound supermassive binary black holes (SBBH) is a natural consequence of the process of merging galaxies containing supermassive black holes in their centres, and the subsequent orbital evolution of the supermassive black holes to the centre of the newly formed galaxy due to dynamic friction. However, when the distance between these black holes becomes sufficiently small, order of $0.01 - 1$ pc, dynamic friction becomes inefficient, since the stars efficiently interacting with the SBBH are scattered off by the binary to hyperbolic orbits. On the other hand, at these distances the loss of orbital energy due to emission of gravitational waves (GW) is typically too small to lead to the merging of the system in Hubble time. The problem of the formation of sources of GW emission due to the SBBH merger was called "the central parsec problem." We propose a new possible solution to this problem determined by a potential non-sphericity of the central star cluster (CSC), which can influence the orbital evolution of the SBBH.

In our analysis we assume that SBBH of unequal mass ratio can reach the distances smaller than the radius of influence of the more massive black hole, where it becomes gravitationally bound. At such distances it has an approximately Keplerian orbit that slowly evolves due to the influence of external perturbations caused by a non-spherical part of the gravitational potential the central star cluster (CSC) and other perturbing factors. It is shown that the orbit can acquire large eccentricities as a result of such evolution, corresponding to periastra order of a few gravitational radii of the more massive black hole. In this case, for certain parameters of the the problem, bursts of GW emission during the periastron passages are efficient enough to cause the loss of the orbital energy sufficient for the SBBH merger in Hubble time.

1. Basic assumptions and the main results

As we've mentioned above it is assumed that the mass ratio of the black holes is small. It is also implied that the distribution of the stellar mass inside the radius of influence of the more massive black hole obeys the standard Young's law. In this case, it is expected that the characteristic size of the orbit (say, its semi-major axis) will quickly decrease due to the influence of dynamic friction down to the radius within which the initial mass of the stars is a quantity of the order of the mass of the lighter black hole. Then, it is also expected that, during several dynamic times, the stars close to the orbit will be ejected from the central cluster by the action of the non-stationary gravitational field of SBBH, and the orbital evolution due to dynamic friction will be stalled. A non-sphericity of the part of CSC external with respect to the orbit of the SBBH is considered in the framework of the quadrupole approximation. In this approximation, generally speaking, the non-spherical part of gravitational potential has two significant independent components determined by spherical harmonics with the azimuthal numbers $m = 0$ and 2 . The first component leads to the usual evolution of von Zeipel-Lidov-Kozai type, where a change of angular momentum of the orbit and, accordingly, its eccentricity is constrained by the requirement that the projection of the angular momentum onto the symmetry plane of the orbit is conserved during the secular evolution. In our work we mainly consider the secular orbital evolution due to the presence of the second component of the perturbing potential having the azimuthal number $m = 2$. In the course of such evolution all three components of the angular momentum are not conserved and the orbit can, in principle, become so elongated, that the effects associated with the presence of the GW emission are significant. Since the secular equations, describing the orbital evolution of such a type, do not have any integrals of motions apart from the Hamiltonian, for their analytical analysis we propose a new method of "quasi-conserved integrals of motion." It is based on the observation that one can find exact solutions to the secular equations corresponding to the singular initial value of the orbital inclination, i_0 , tending to zero at the initial time $t = 0$, $i_0 \rightarrow 0^+$. We also assume, for simplicity, that the initial eccentricity e_0 is small

but finite. These 'singular' solutions have the property that the eccentricity e becomes equal to one at some particular time. They can be continued to the later times using the requirement that the nodal and apsidal angles are continuous at the time corresponding to $e = 1$, while the inclination angle i is discontinuous; it experiences a jump from 0^+ to π^- at this time. The evolution at $i \rightarrow \pi^-$ can again be described analytically. It proceeds until another moment of time corresponding to $e = 1$ is reached, where we again assume that the nodal and apsidal angles are continuous, while i jumps from π^- to 0^+ . These singular solutions have a cyclic character. We then treat non-singular solutions having a small, but finite value of i_0 as perturbations over the singular ones. They have a small, but finite, deviation of the maximal eccentricity from one $\propto i_0$. These non-singular solutions can be described in terms of a slow evolution of the initial values of the dynamical variables describing the singular system, which are, by definition, its integrals of motion, and i_0 . In the same way we can also describe the influence of other perturbing factors, most importantly, the influence of Einstein's precession of the apsidal angle. In general, similar to the singular case, the orbital evolution starting with some small values of eccentricity e_0 and i_0 can be divided on four subsequent stages. At first stage i stays sufficiently close to i_0 , while eccentricity grows to values sufficiently close to 1. We then have a sharp transition of i from the small value to a value close to π . When the inclination is equal to $\pi/2$ during this sharp transition the eccentricity passes through a local maximum. Then the eccentricity goes again to a small value close to e_0 , while the inclination stays close to $\pi/2$. Then the eccentricity starts to grow again, until the next sharp transition of inclination from $\sim \pi/2$ to ~ 0 happens. Then the eccentricity decreases again to a value close to the initial one, while the inclination stays close to the small value. Then the cycle repeats. A distinctive feature of these non-singular solutions is that their parameters, such as e_0 , i_0 , the maximal values of eccentricity are slowly changing from one cycle to another, and the cycles are only quasi-periodic. The numerically obtained evolution of inclination and eccentricity during one quasi-periodic cycle is shown in Fig. 1figure.1. We find some mapping relations, describing the small changes of the parameters over one cycle of quasi-periodic evolution. However, a typical duration time of these cycles is always of the order of the von Zeipel-Lidov-Kozai time period. We have checked that this behaviour is in an excellent quantitative agreement with the numerical results. Moreover, it was numerically demonstrated that when the evolution starts with values of e_0 and i_0 order of one, it has some transient periods when the evolution is qualitatively similar to the one described above. When the Einstein precession is significant enough, it may be shown using the arguments based on the conservation of the Hamiltonian of the dynamical system that the deviation of a maximal value of eccentricity from one over many cycles of the evolution is proportional to its typical rate. This result allows us to estimate a typical value of the periastron distance and the role of GW emission on the orbital evolution and find the parameters of SBBH and the star cluster corresponding to SBBH merger over one Hubble time. A distinctive feature of GW sources formed as a result of the action of our effect is that the eccentricity

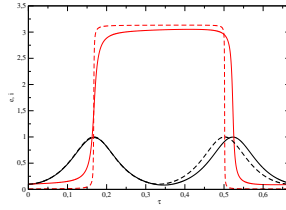


FIGURE 1. The eccentricities e (black lines) and inclinations i (red lines) shown as functions of the 'slow' time τ , which is chosen in such a way, that one quasi-period has always a value of τ order of one. Solid and dashed lines have $i_0 = 0.1$ and 0.01 , respectively.

may remain significant in the course of all stages of the evolution leading to the merger, see [1] for more detail.

Conclusions

We have proposed a new mechanism leading to the solution of the central parsec problem based on a potential non-sphericity of the central star cluster. The main attention was paid to the perturbing gravitational potential with $m = 2$ leading to a non-standard secular evolution. This evolution was treated analytically and numerically. It was shown that the evolution has a quasi-cyclic character and the Einstein precession is, for typical parameters of SBBH and the cluster, the main effect limiting a value of eccentricity over many (say, ~ 100) cycles. We estimated the parameters of SBBH and the cluster corresponding to SBBH merger in one Hubble time.

References

- [1] P. B. Ivanov, A. G. Polnarev, *Physical Review D*, Vol. 112, ID. 043030, (2025).

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